ECCHacks:
a gentle introduction
to elliptic-curve cryptography

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University of Illinois at Chicago &
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Technische Universiteit Eindhoven

`ecchacks.cr.yp.to`

Cryptography

Public-key signatures:
e.g., RSA, DSA, ECDSA.
Some uses: signed OS updates,
SSL certificates, e-passports.

Public-key encryption:
e.g., RSA, DH, ECDH.
Some uses: SSL key exchange,
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Secret-key encryption:
e.g., AES, Salsa20.
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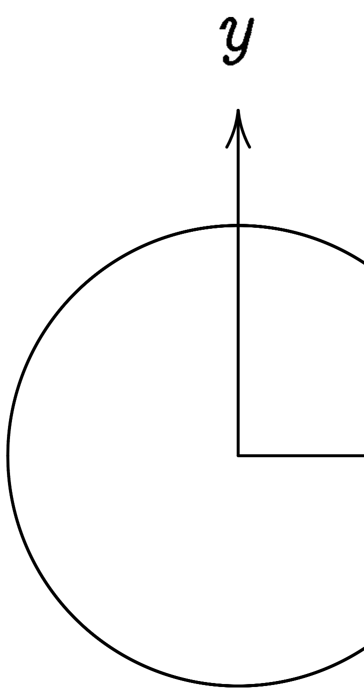

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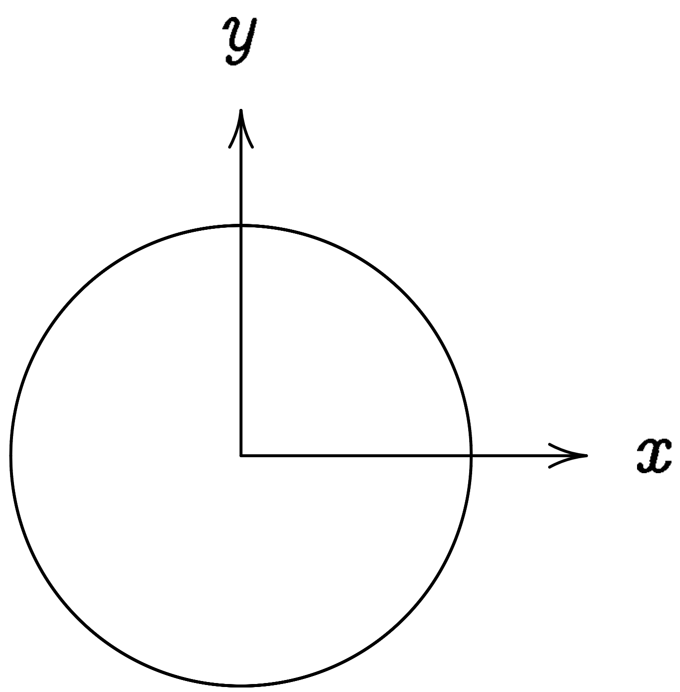

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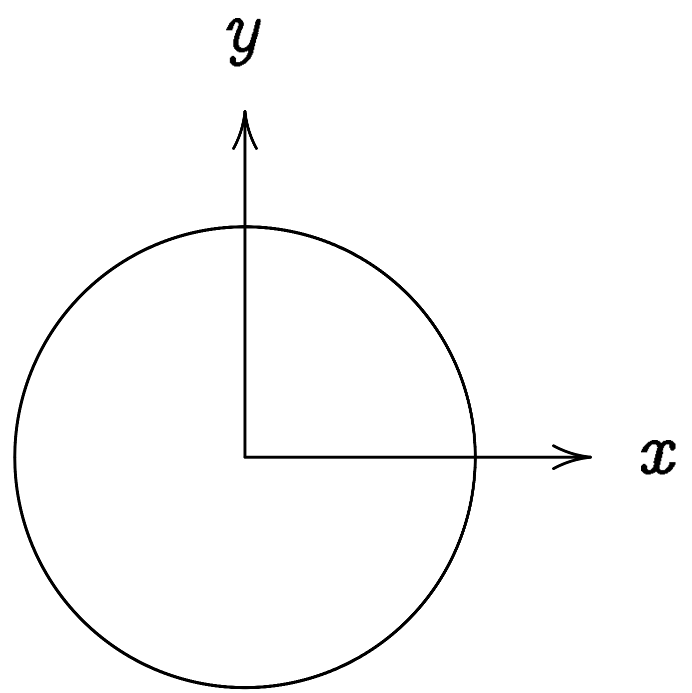

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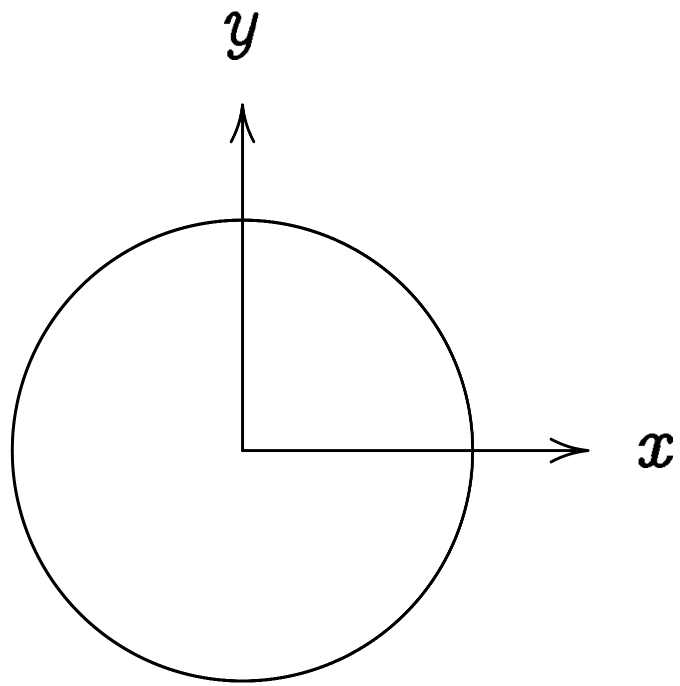

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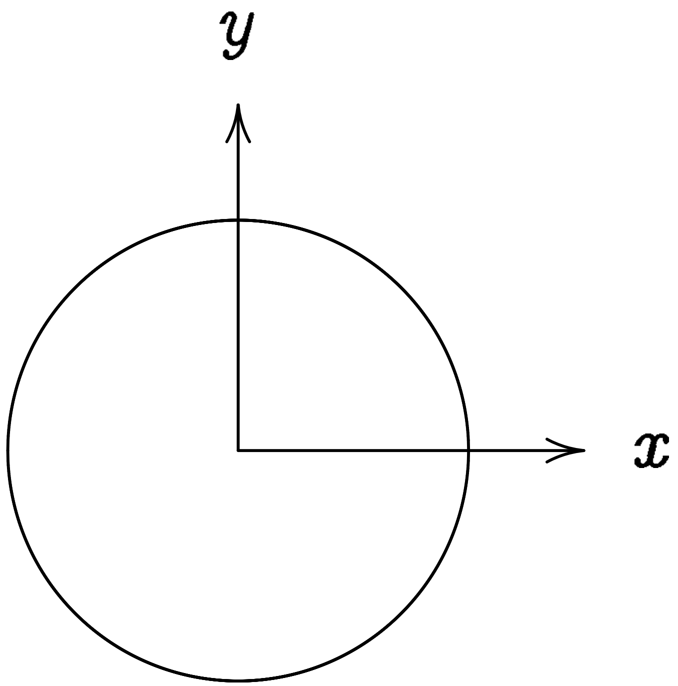

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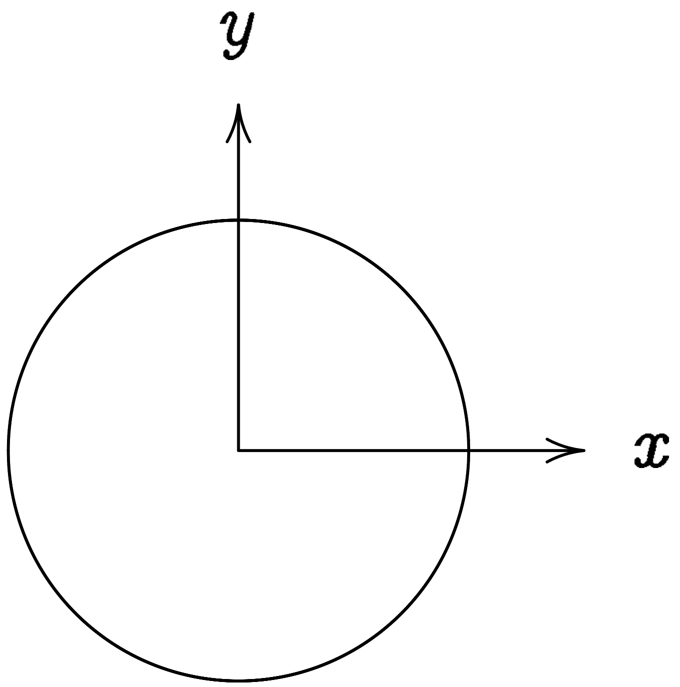

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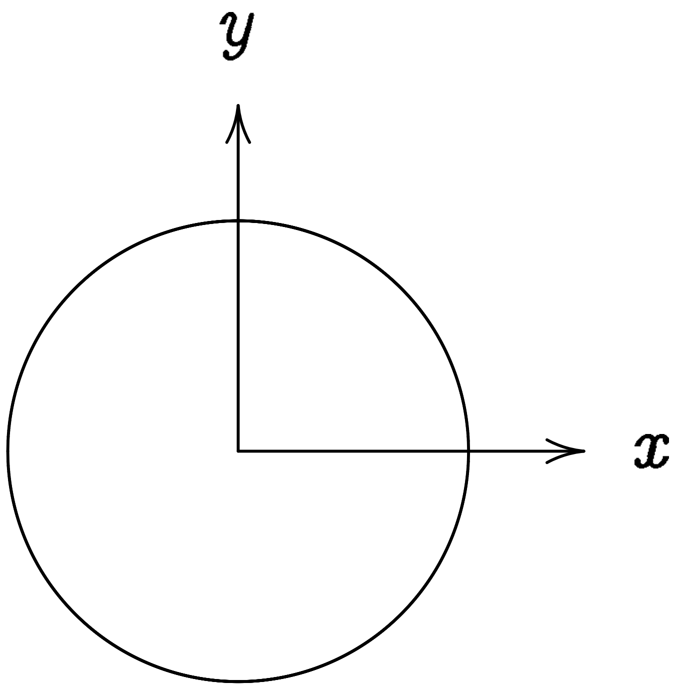

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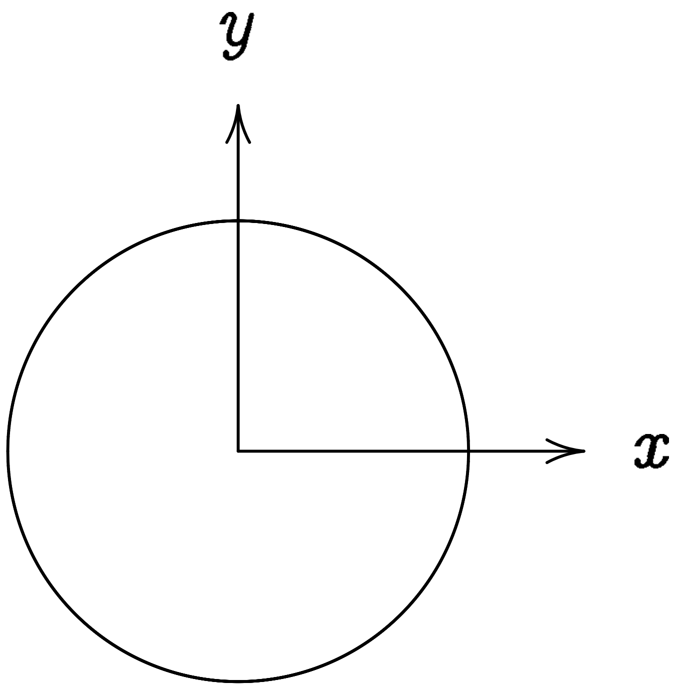

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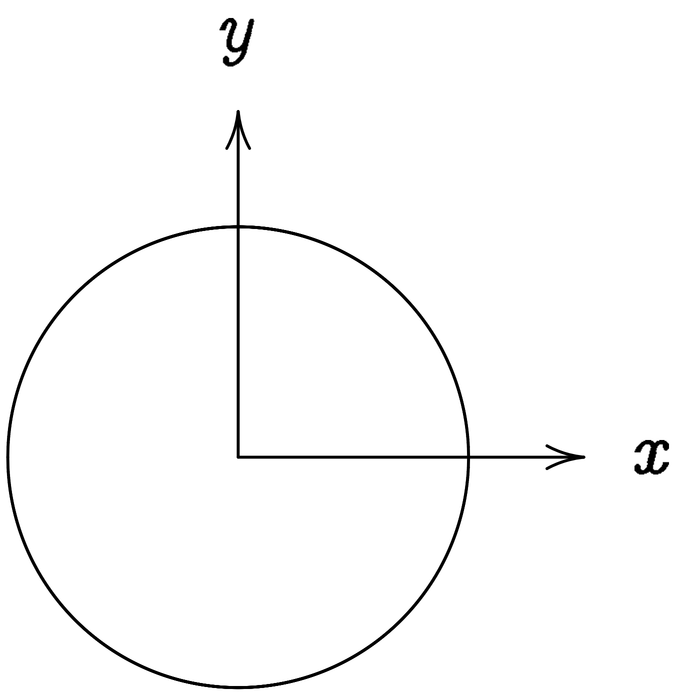

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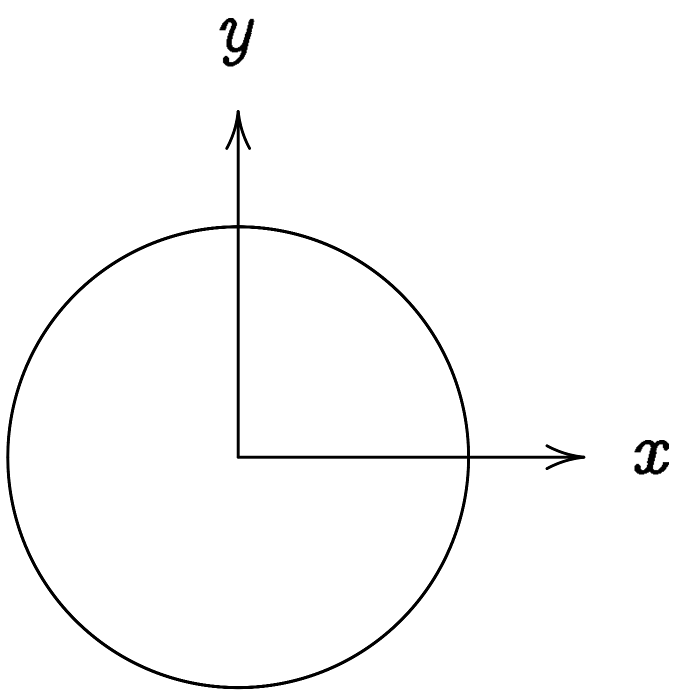

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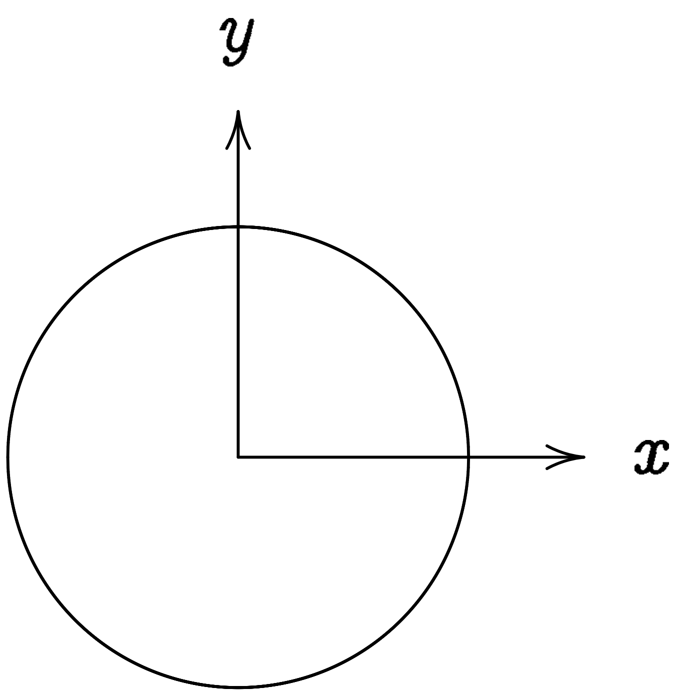

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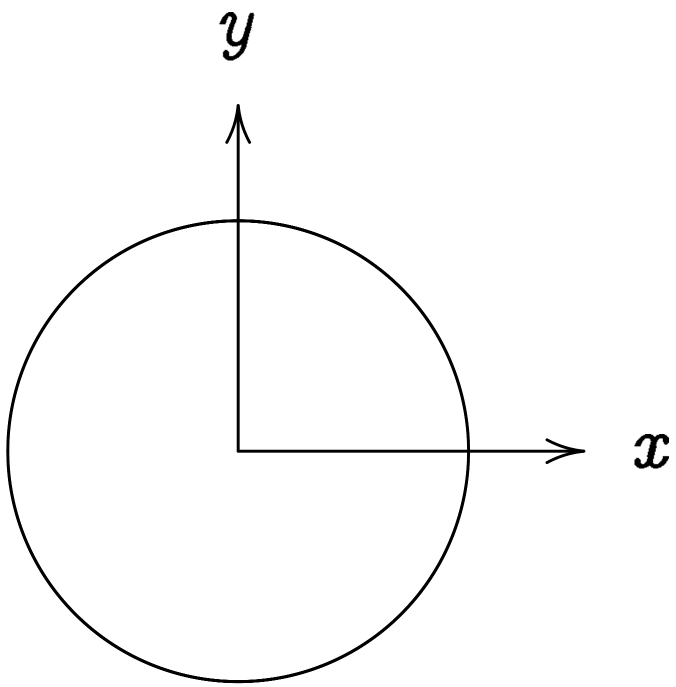

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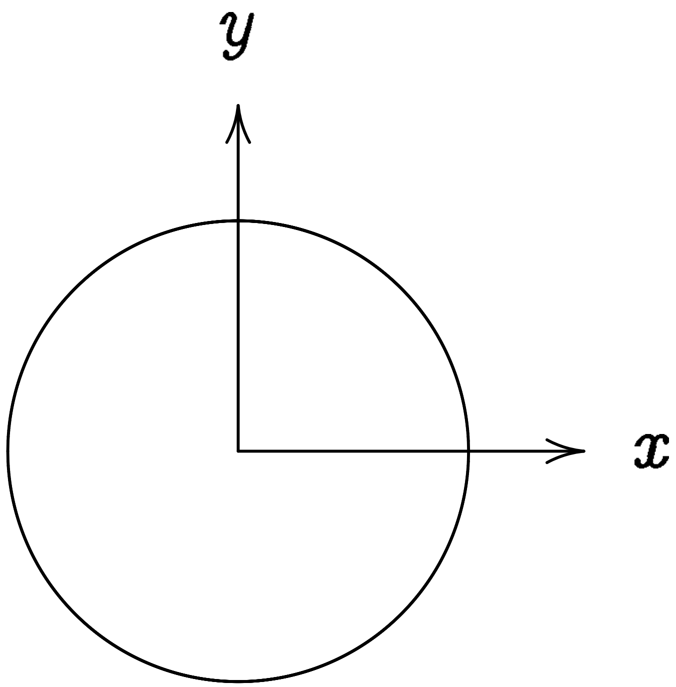

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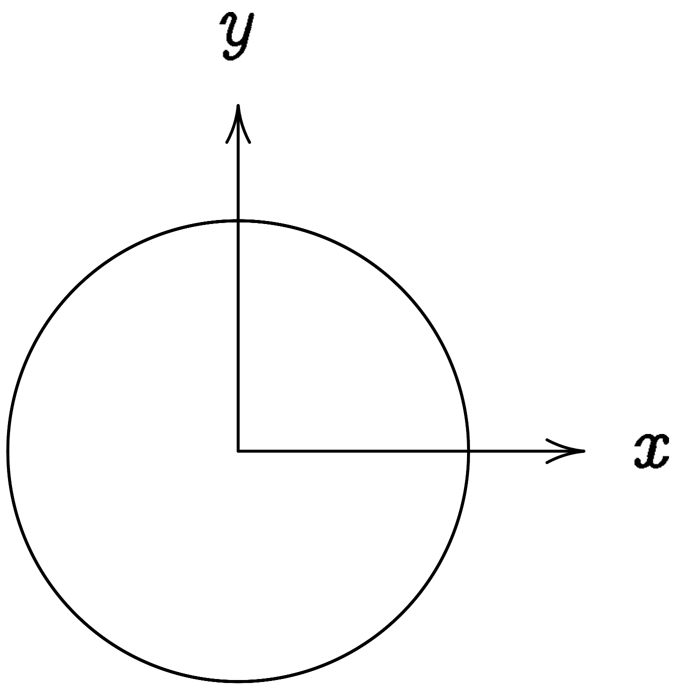

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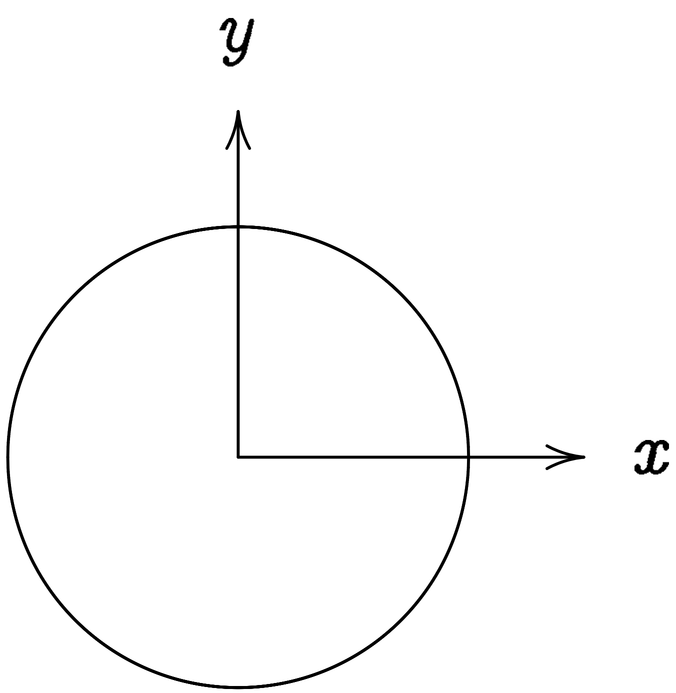

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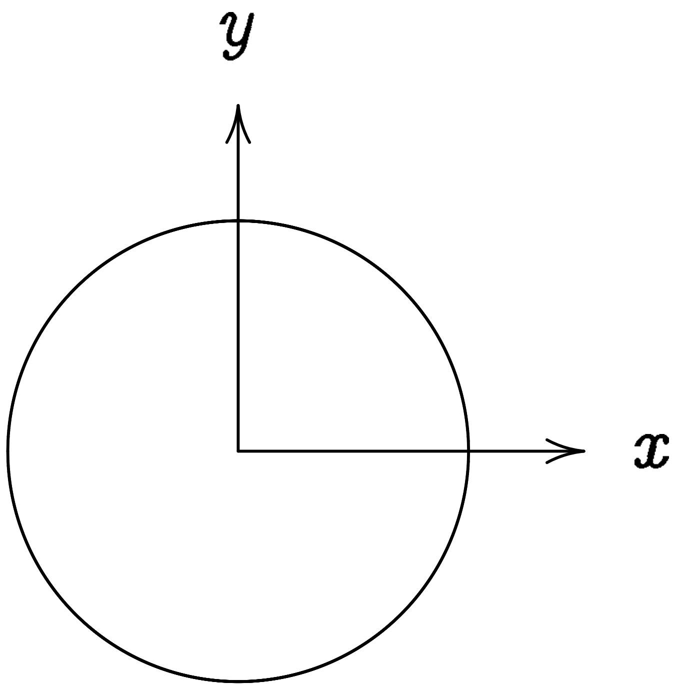

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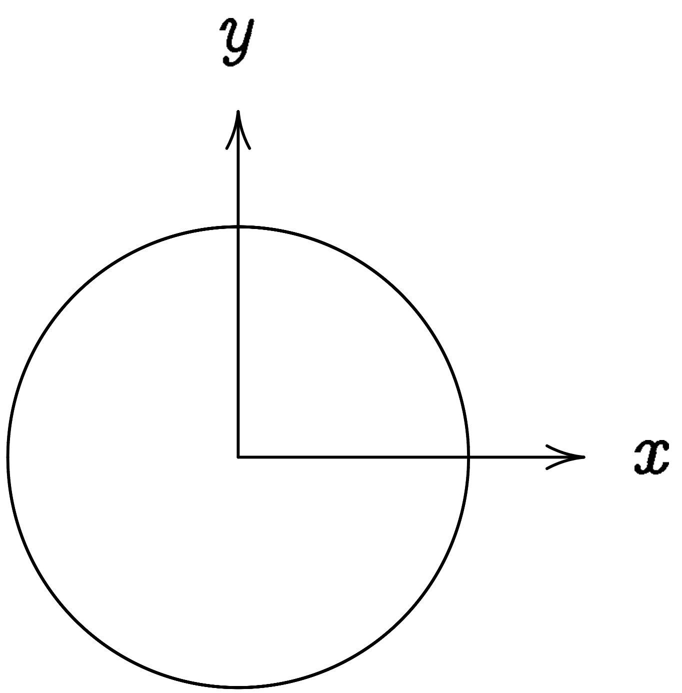

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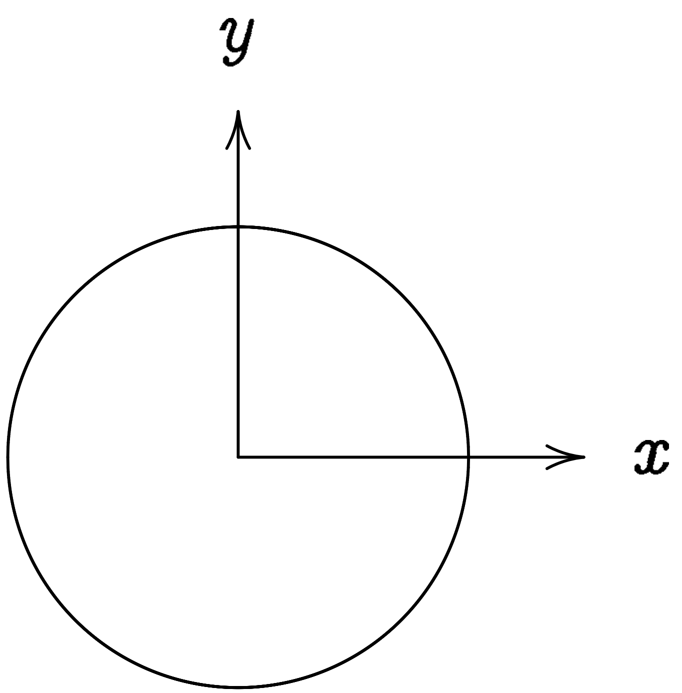

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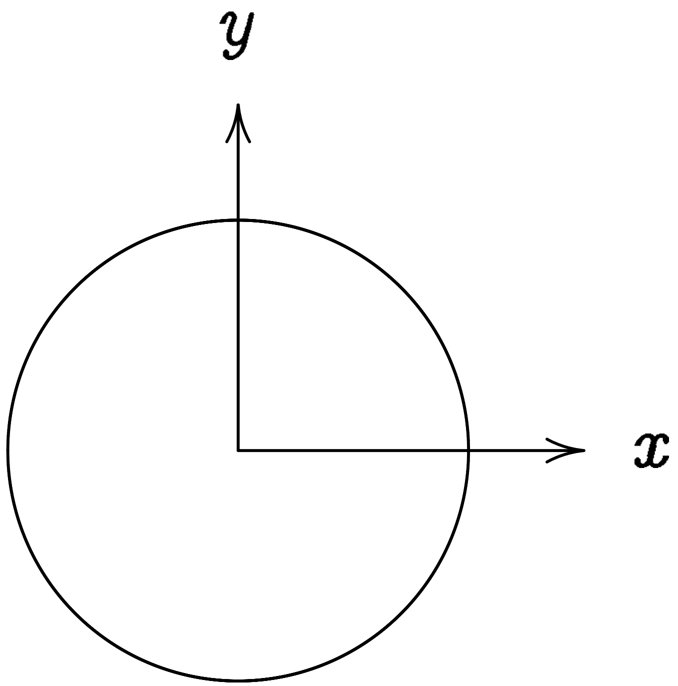

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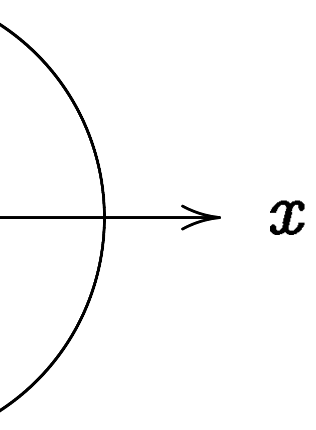

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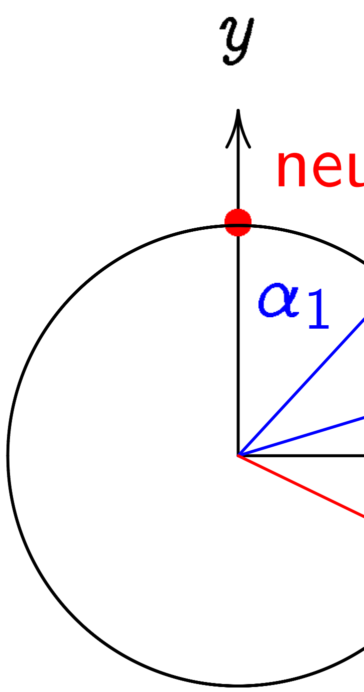

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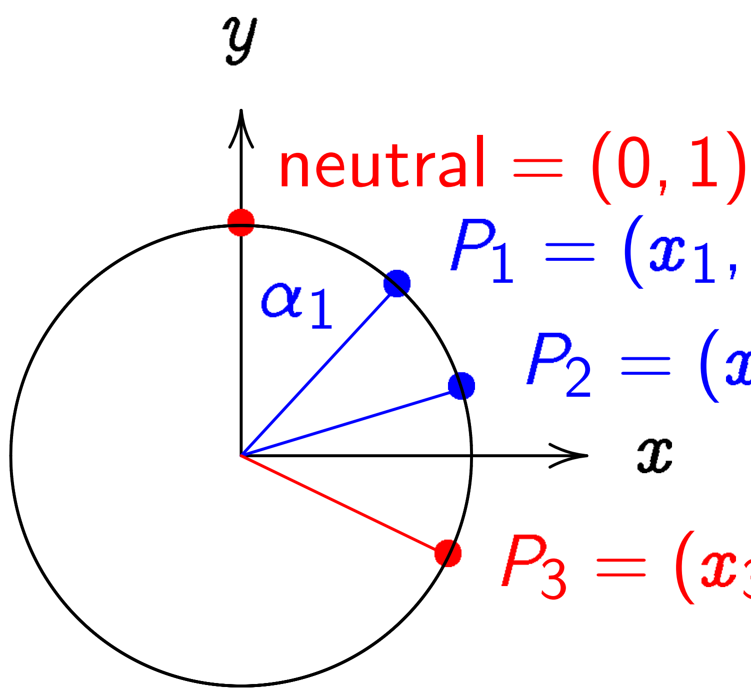
Addition on the clock:



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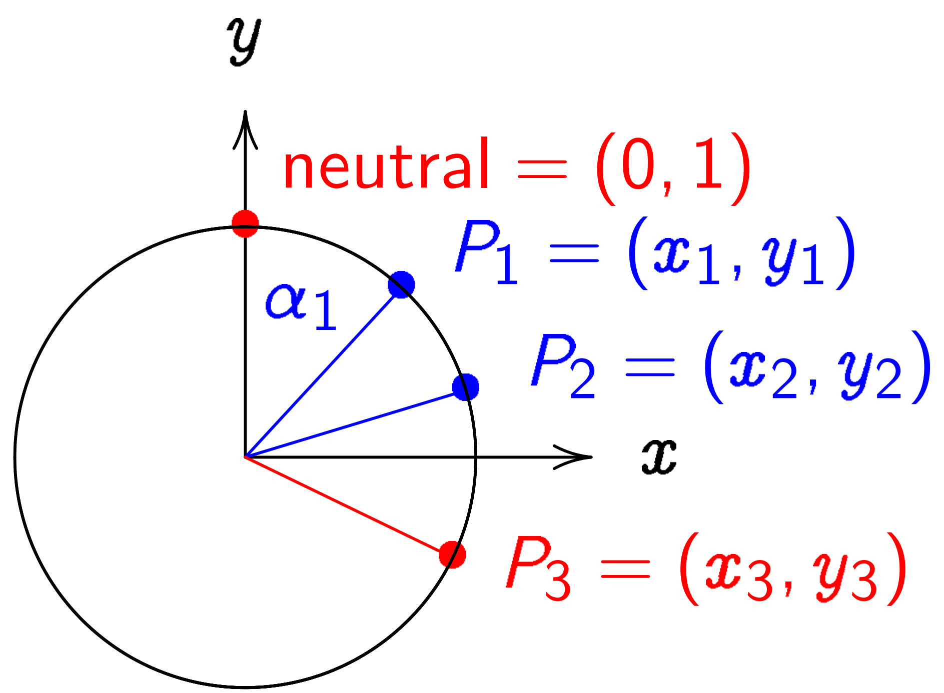

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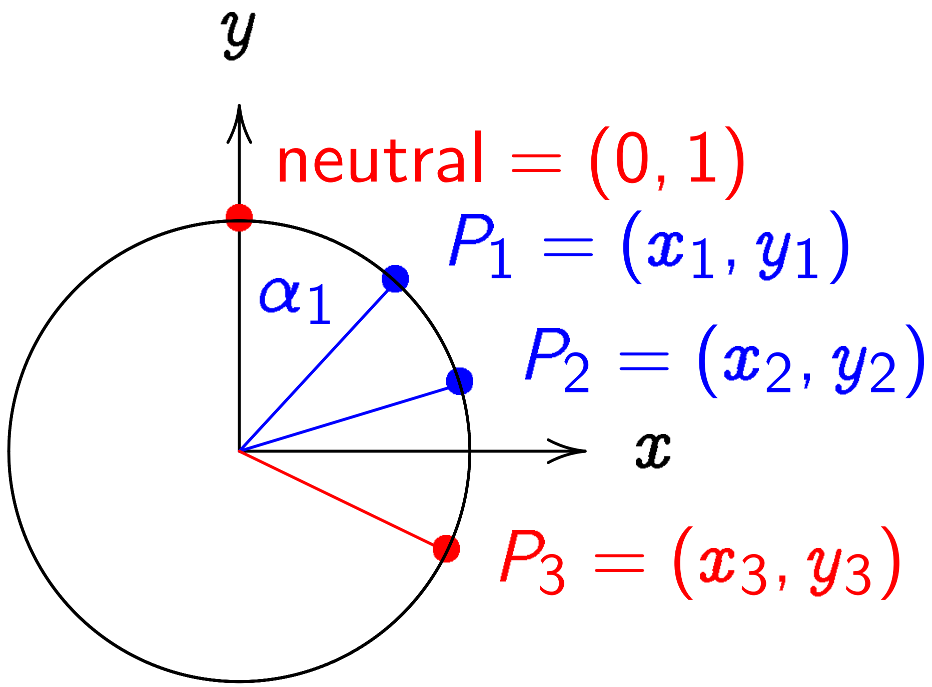

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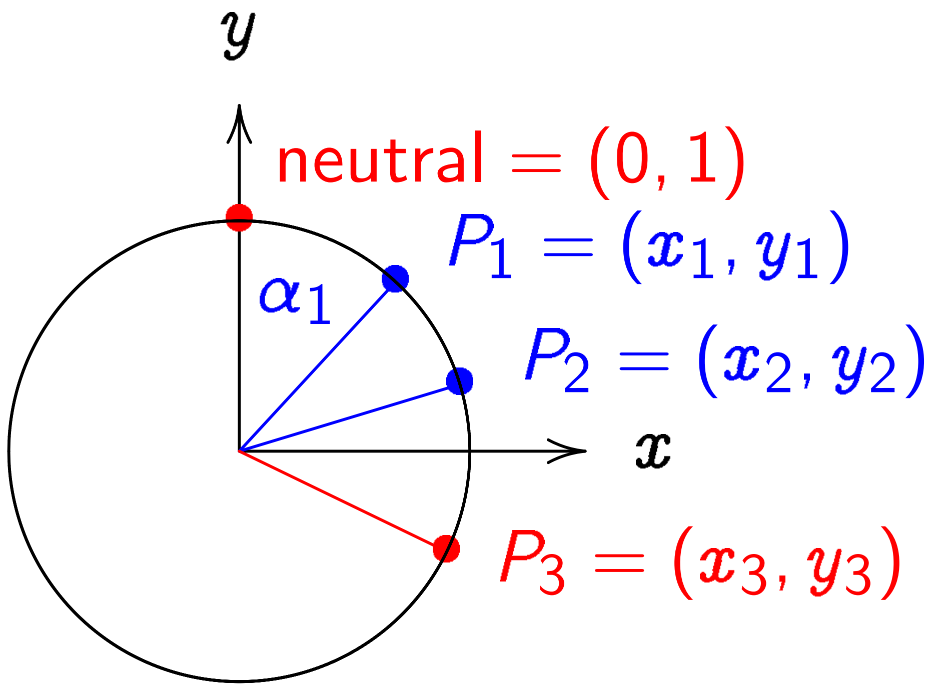

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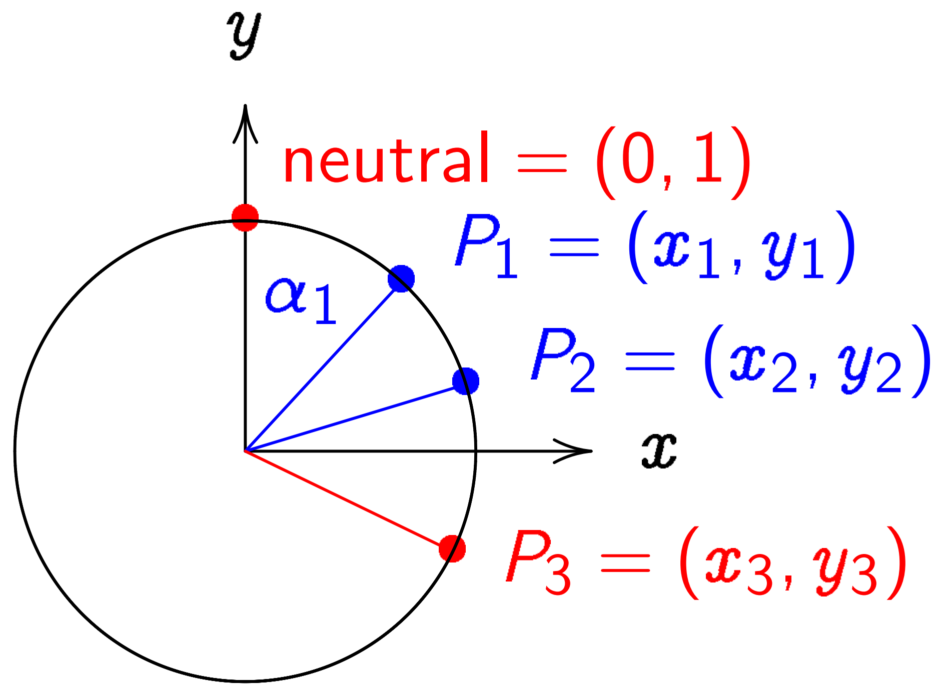

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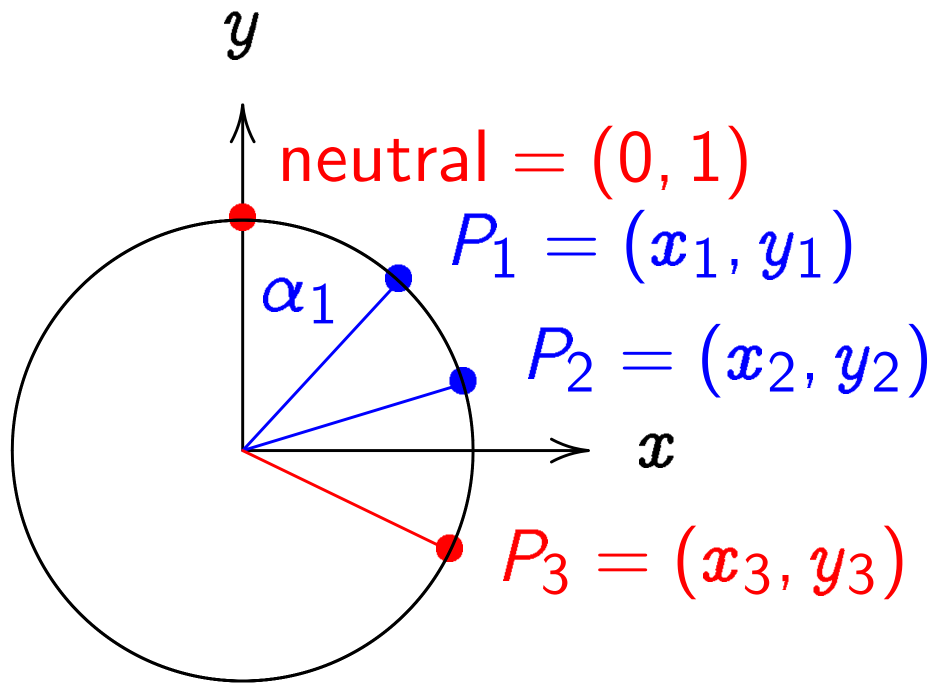

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Clock addi

Use Cartes

Addition fo

for the cloc

sum of $(x_1$

$(x_1 y_2 + y_1$

this curve:

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3/5).

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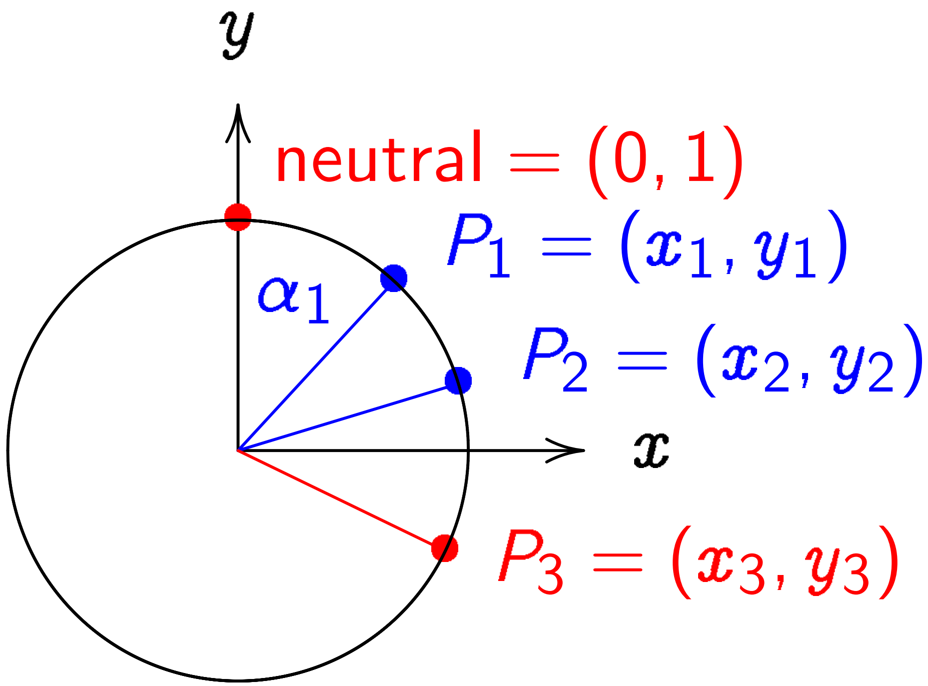

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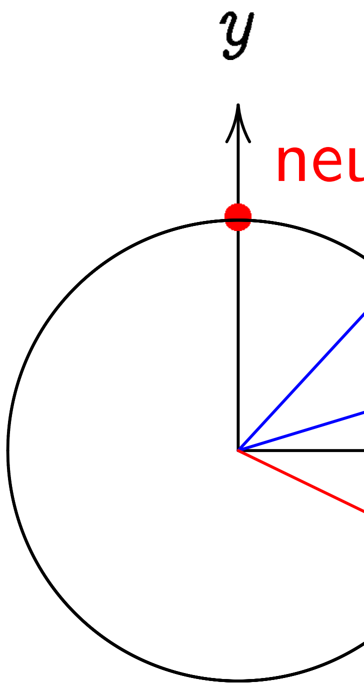

Use Cartesian coordinat

Addition formula

for the clock $x^2 + y^2 =$

sum of (x_1, y_1) and $(x_2$

$(x_1 y_2 + y_1 x_2, y_1 y_2 - x$

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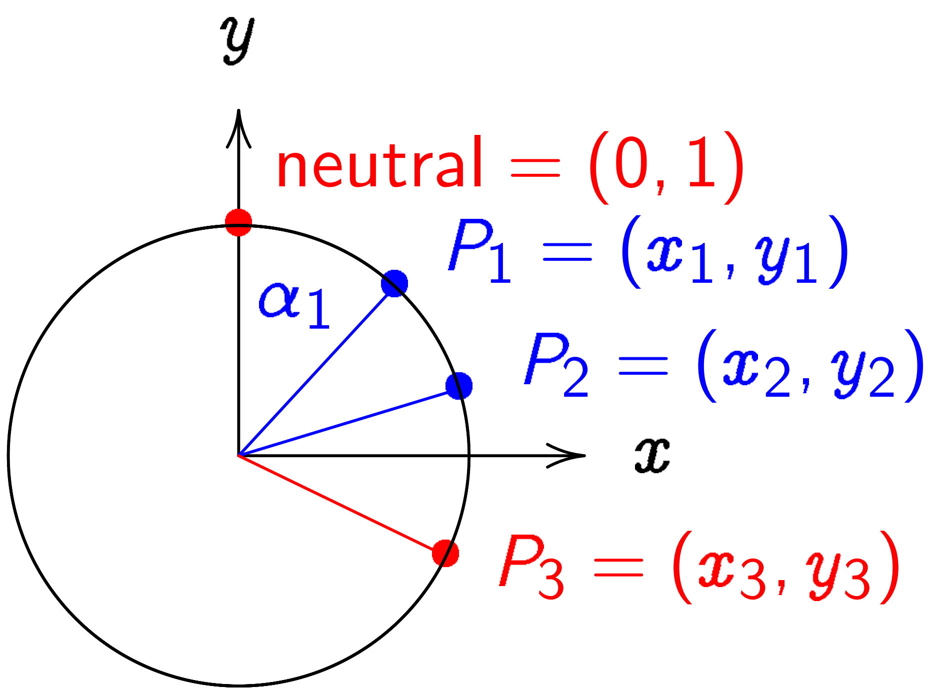

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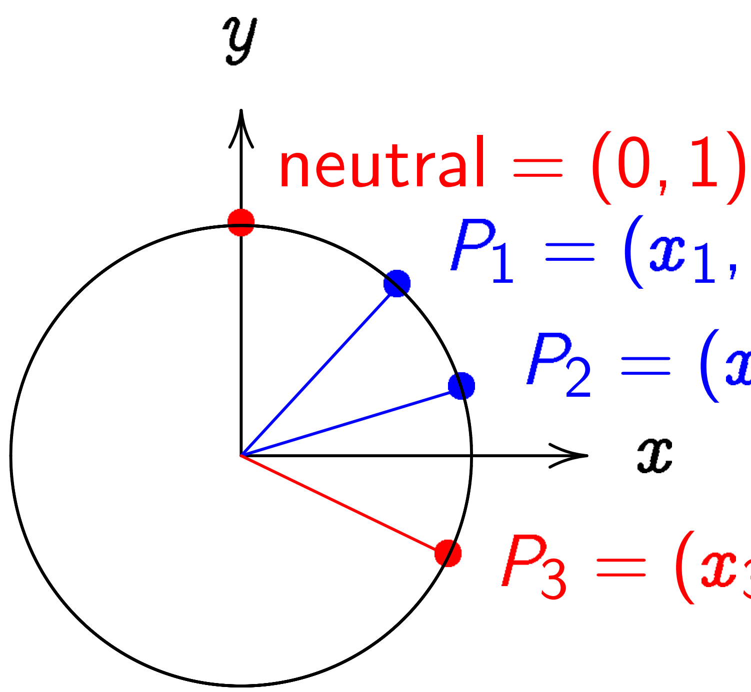

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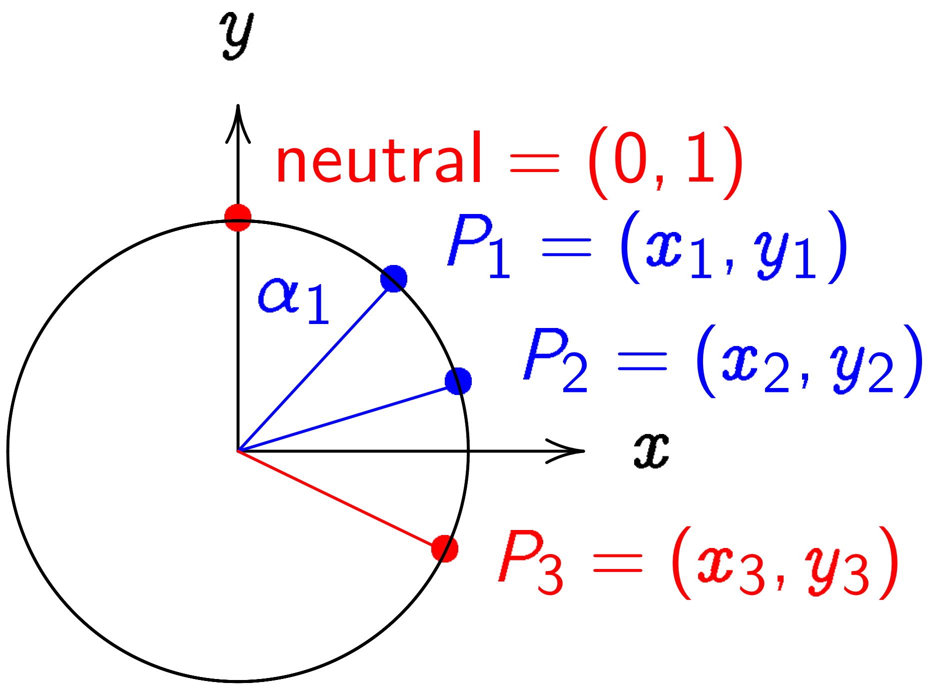

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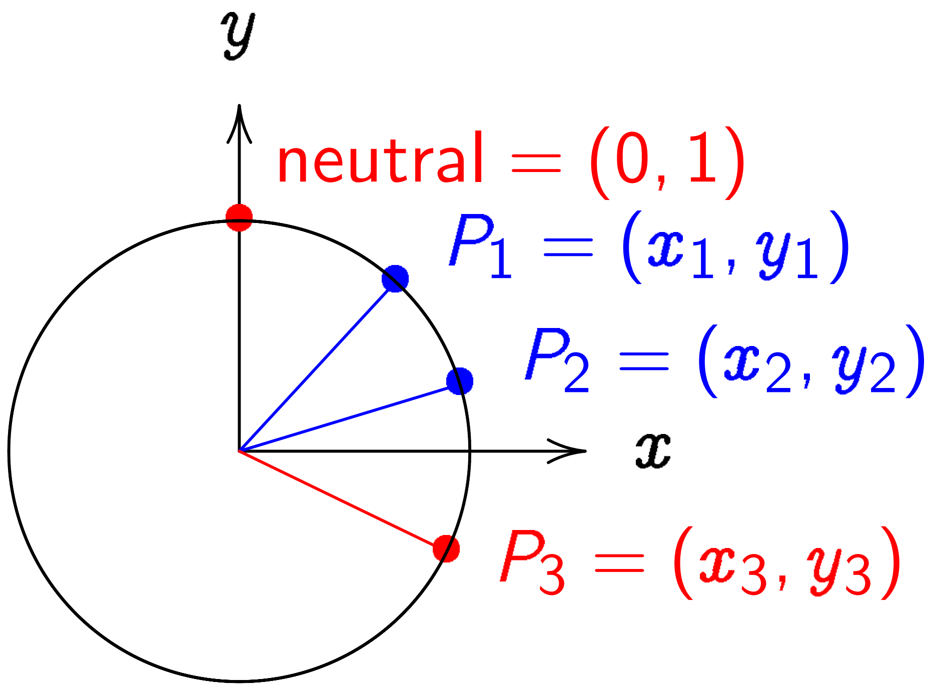

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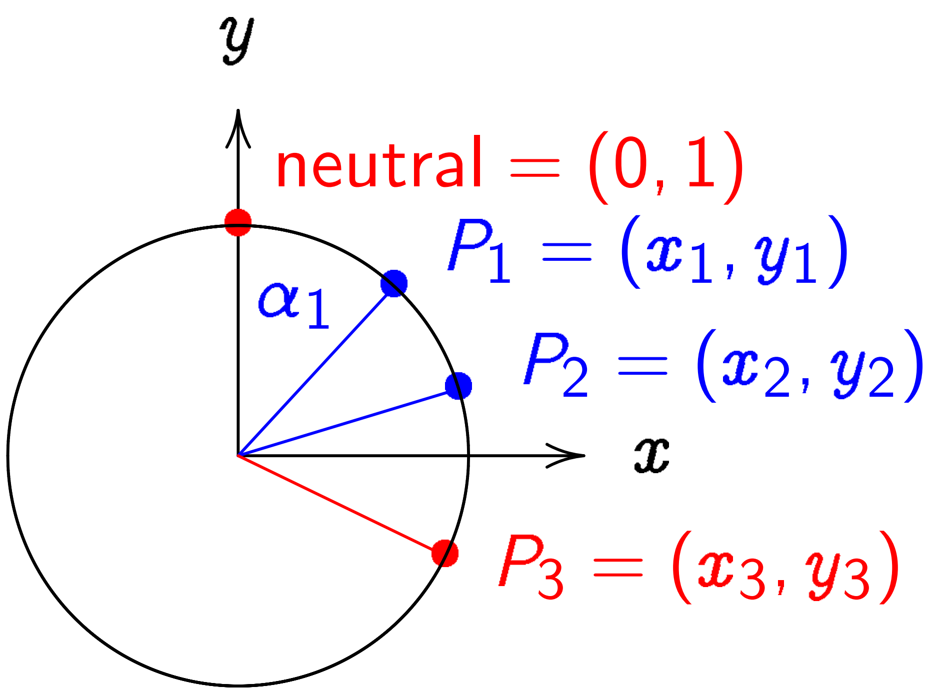

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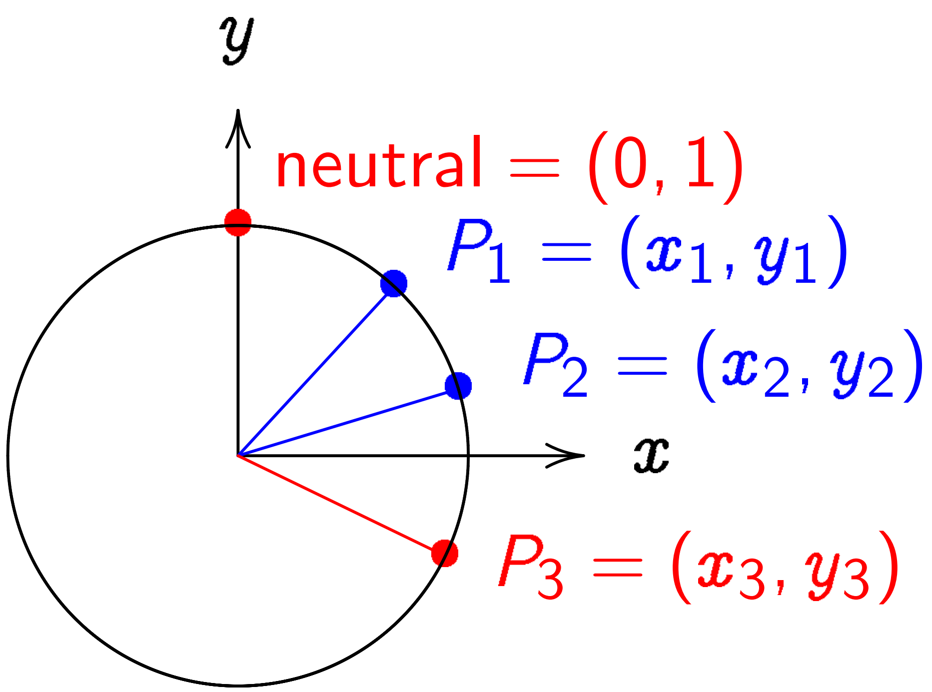

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Examples c

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"5:00" + "

$= (1/2, -\sqrt{}$

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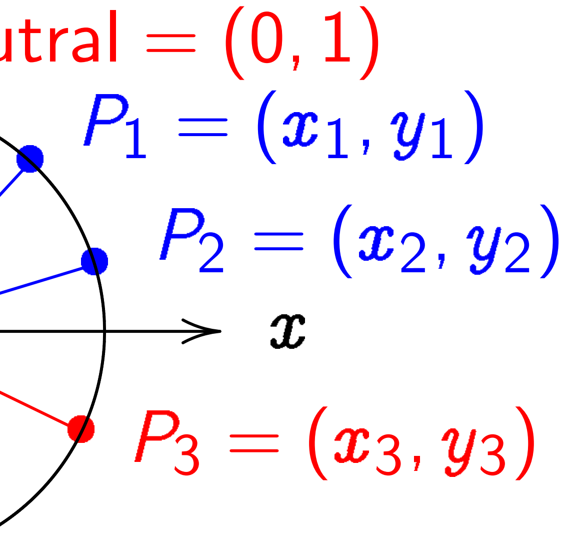
$2\left(\frac{3}{5}, \frac{4}{5}\right) =$



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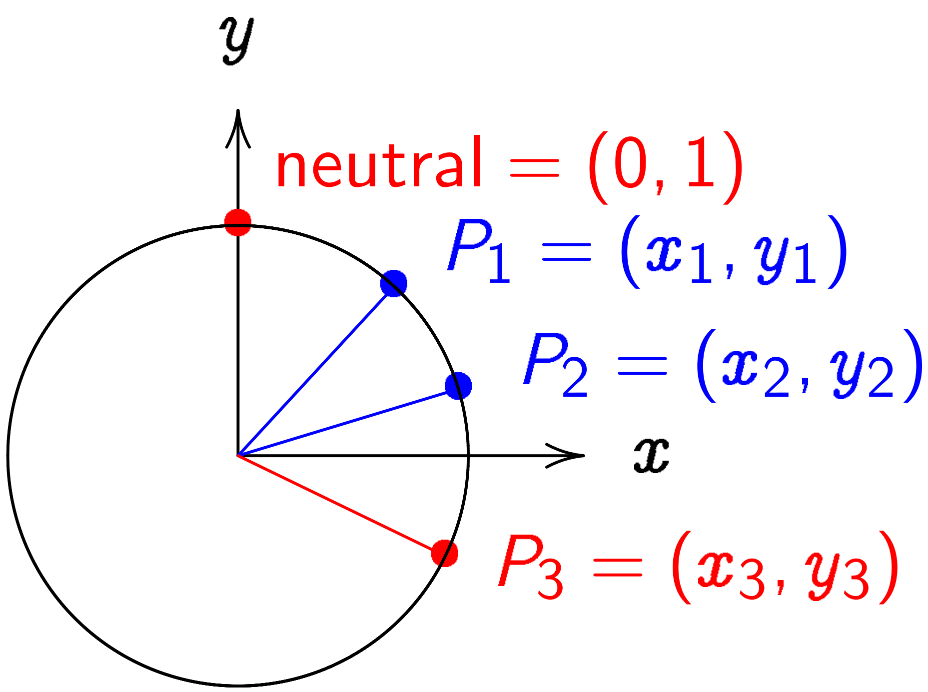

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$y_1)$

$x_2, y_2)$

$_3, y_3)$

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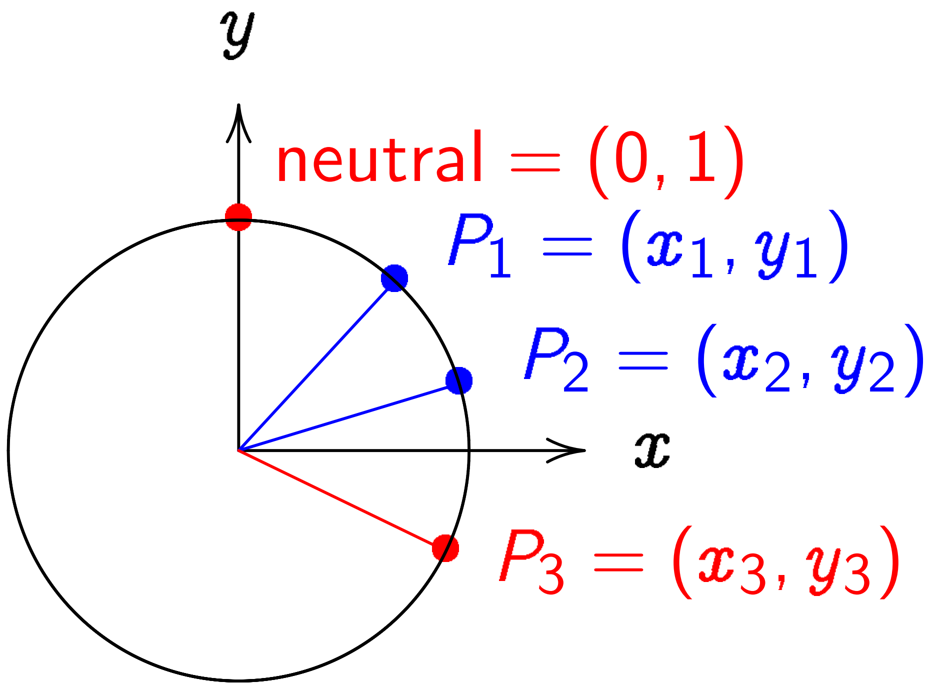

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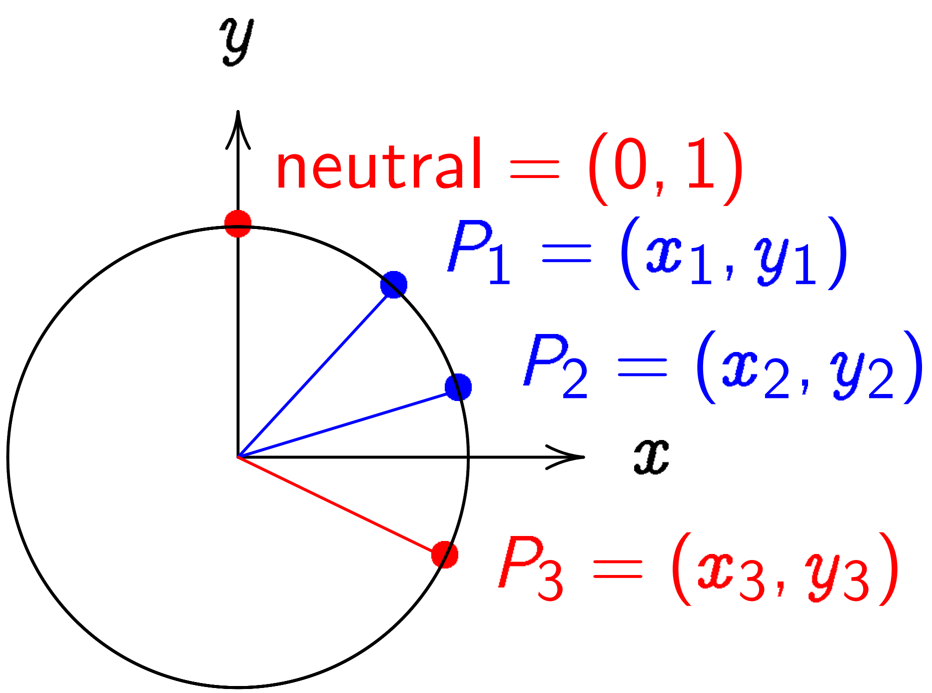

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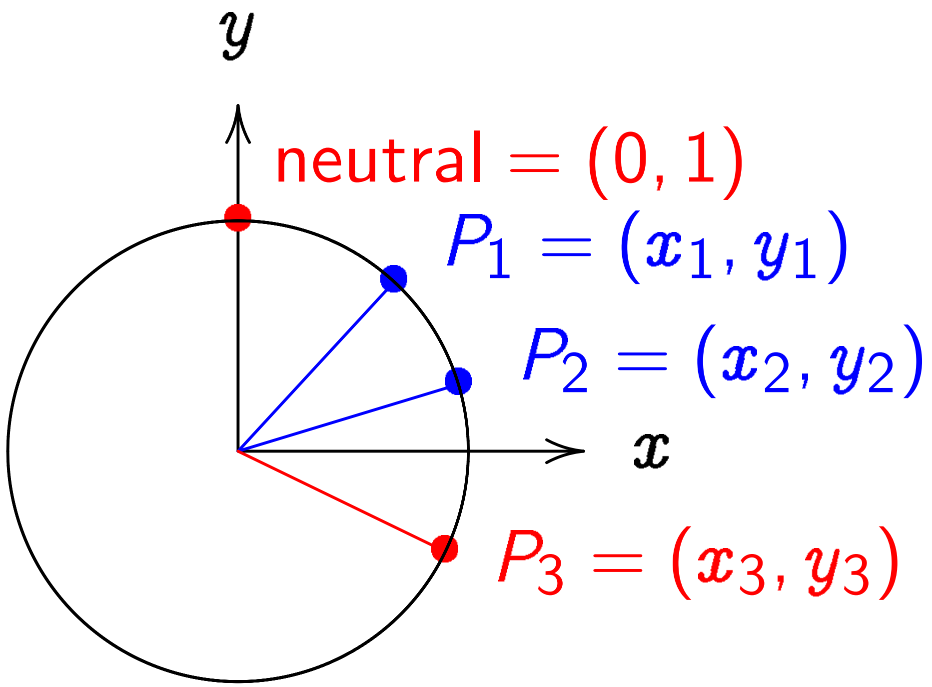

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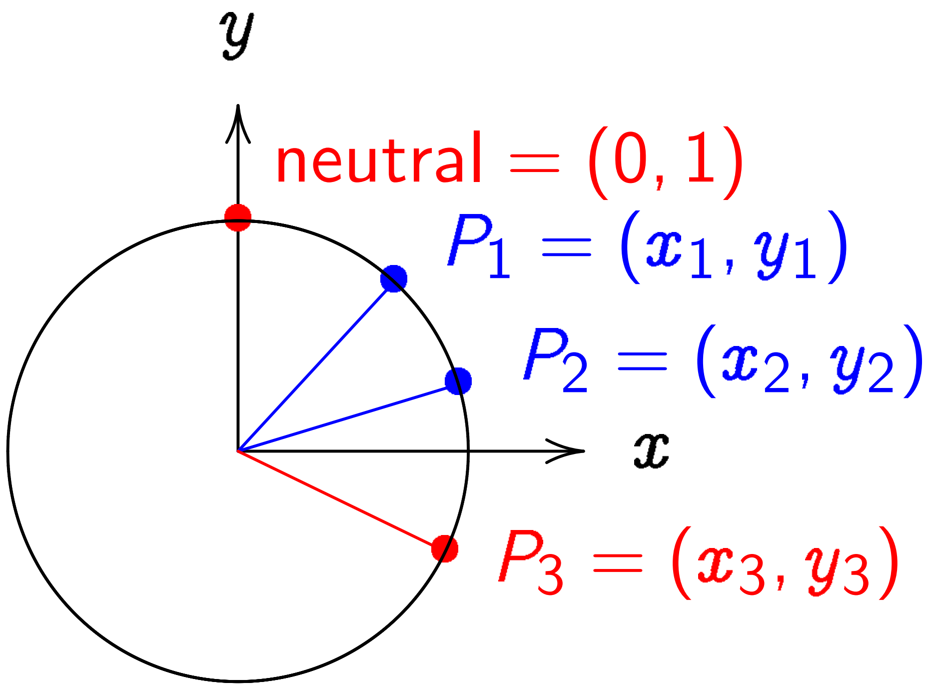

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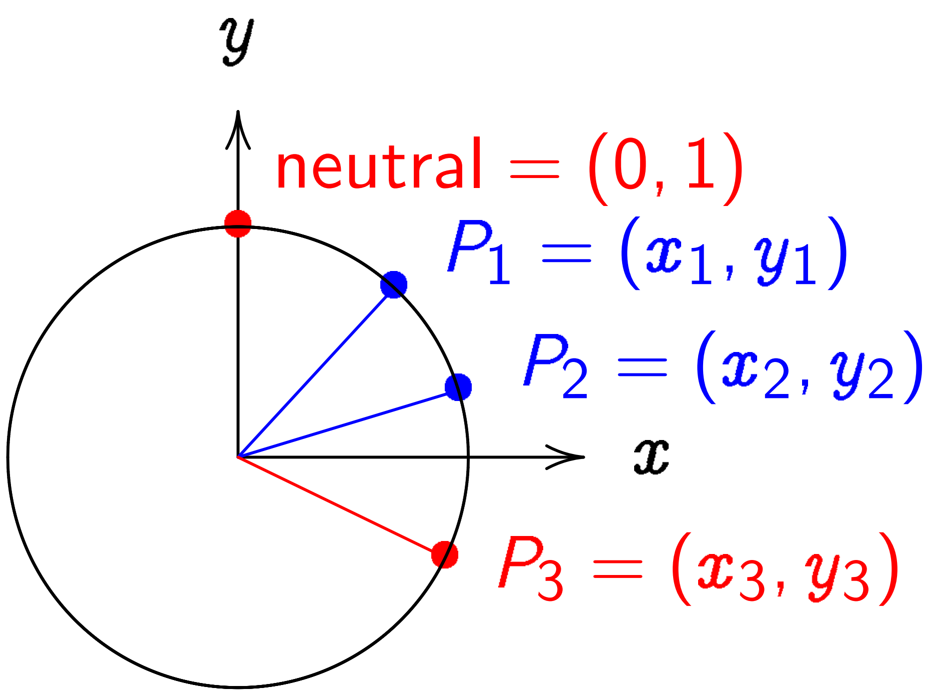

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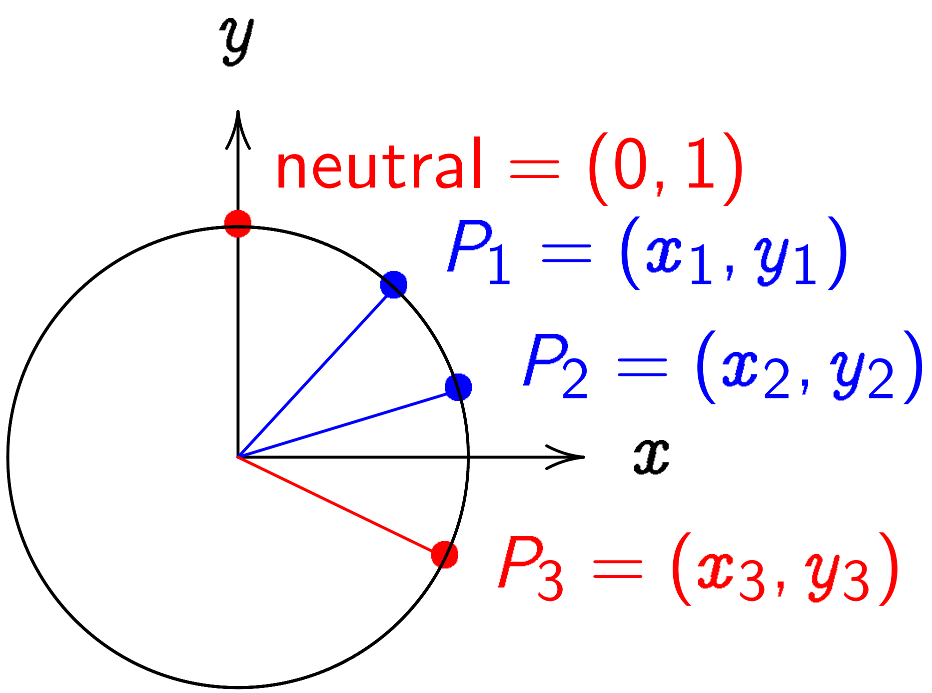

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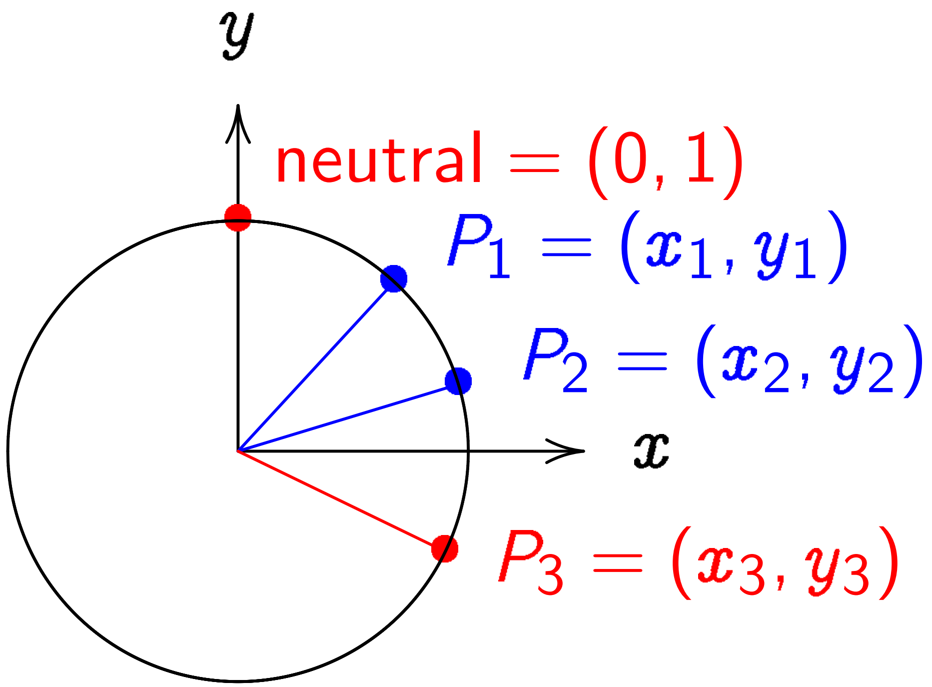

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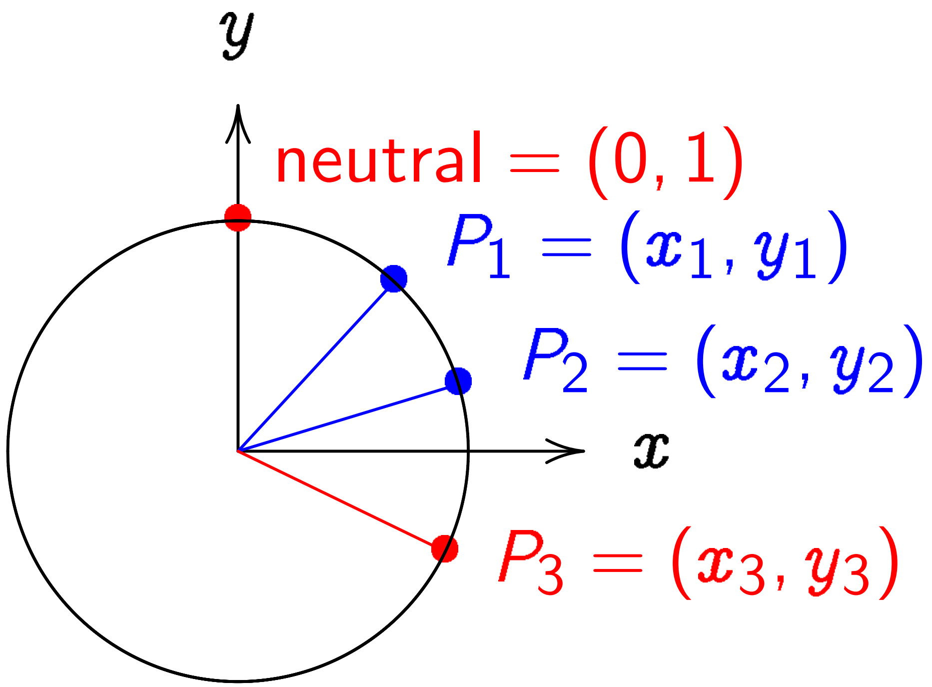

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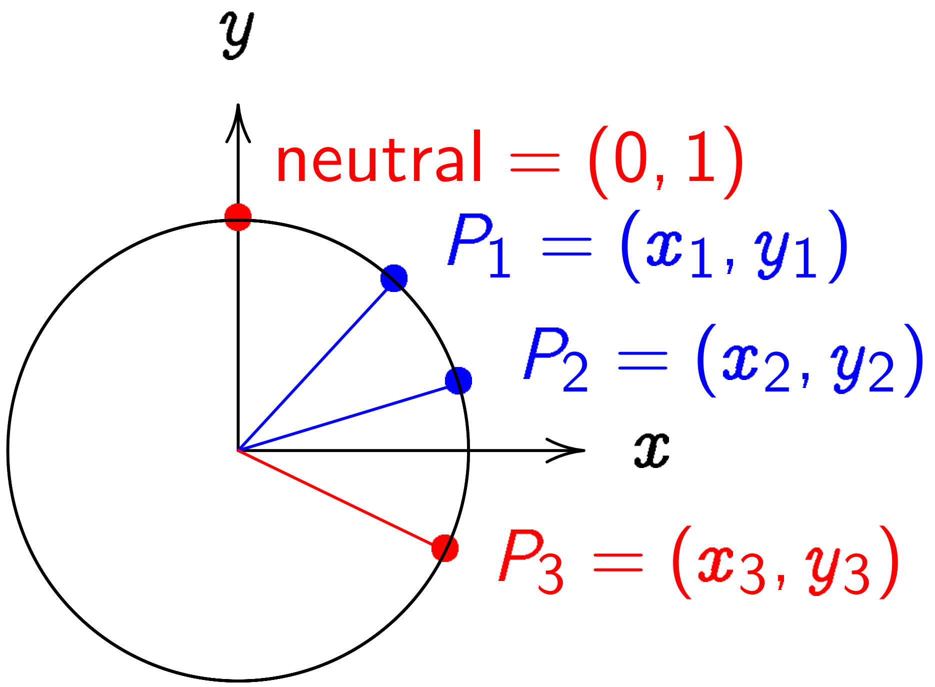
$$3\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{117}{125}, \frac{-44}{125}\right).$$

$$4\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{336}{625}, \frac{-527}{625}\right).$$

$(x_1, y_1) + (0, 1) = (x_1, y_1)$.

$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

tion without sin, cos:



ian coordinates for addition.

ormula

ck $x^2 + y^2 = 1$:

, $y_1)$ and (x_2, y_2) is

$x_2, y_1 y_2 - x_1 x_2)$.

Examples of clock addition:

"2:00" + "5:00"

$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$

$= (-1/2, -\sqrt{3/4}) =$ "7:00".

"5:00" + "9:00"

$= (1/2, -\sqrt{3/4}) + (-1, 0)$

$= (\sqrt{3/4}, 1/2) =$ "2:00".

$$2\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{24}{25}, \frac{7}{25}\right).$$

$$3\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{117}{125}, \frac{-44}{125}\right).$$

$$4\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{336}{625}, \frac{-527}{625}\right).$$

$(x_1, y_1) + (0, 1) = (x_1, y_1)$.

$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

Clocks ove

Clock($\mathbf{F}_7$)

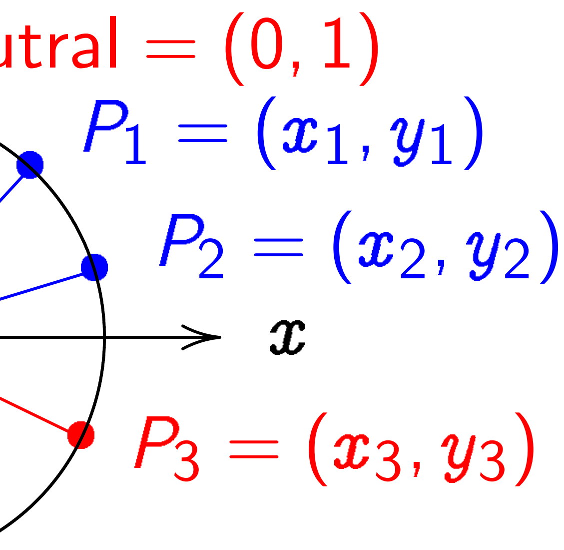
Here $\mathbf{F}_7 =$

$=$

with arithm

e.g. $2 \cdot 5 =$

sin, cos:



tes for addition.

= 1:

$_2, y_2)$ is

$_1 x_2)$.

Examples of clock addition:

"2:00" + "5:00"

$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$

$= (-1/2, -\sqrt{3/4}) =$ "7:00".

"5:00" + "9:00"

$= (1/2, -\sqrt{3/4}) + (-1, 0)$

$= (\sqrt{3/4}, 1/2) =$ "2:00".

$$2\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{24}{25}, \frac{7}{25}\right).$$

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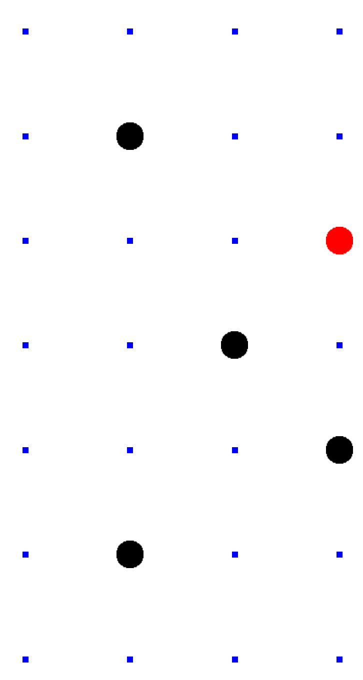
$(x_1, y_1) + (0, 1) = (x_1, y_1)$.

$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

Clocks over finite fields



$\mathrm{Clock}(\mathbf{F}_7) = \{(x, y) \in$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4,$

$= \{0, 1, 2, 3, -$

with arithmetic modulo

e.g. $2 \cdot 5 = 3$ and $3/2 =$

$y_1)$

$x_2, y_2)$

$3, y_3)$

on.

Examples of clock addition:

"2:00" + "5:00"

$$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$$
$$= (-1/2, -\sqrt{3/4}) = \text{“7:00”}.$$

"5:00" + "9:00"

$$= (1/2, -\sqrt{3/4}) + (-1, 0)$$
$$= (\sqrt{3/4}, 1/2) = \text{“2:00”}.$$

$$2\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{24}{25}, \frac{7}{25}\right).$$

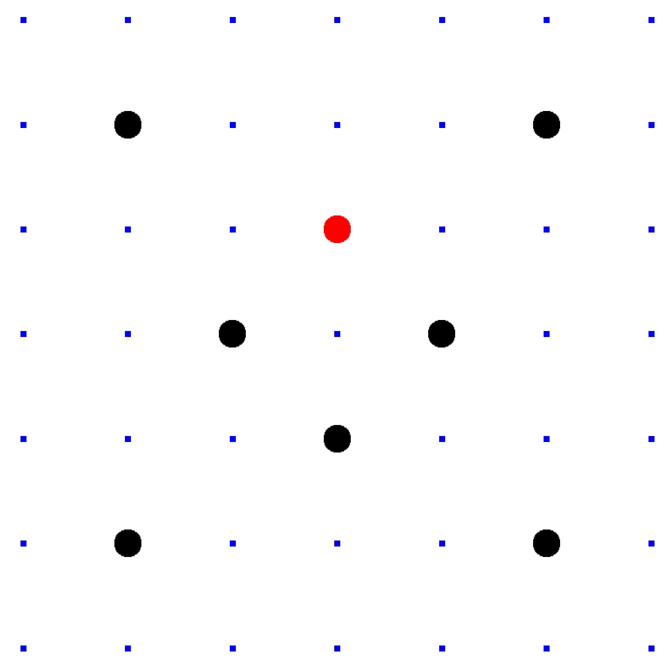
$$3\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{117}{125}, \frac{-44}{125}\right).$$

$$4\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{336}{625}, \frac{-527}{625}\right).$$

$$(x_1, y_1) + (0, 1) = (x_1, y_1).$$

$$(x_1, y_1) + (-x_1, y_1) = (0, 1).$$

Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

Examples of clock addition:

"2:00" + "5:00"
$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$
$= (-1/2, -\sqrt{3/4}) =$ "7:00".

"5:00" + "9:00"
$= (1/2, -\sqrt{3/4}) + (-1, 0)$
$= (\sqrt{3/4}, 1/2) =$ "2:00".

$$2\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{24}{25}, \frac{7}{25}\right).$$

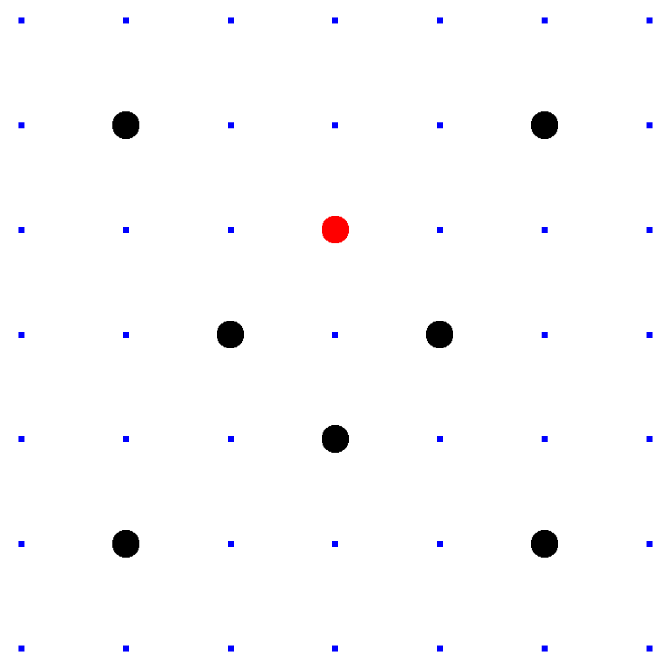
$$3\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{117}{125}, \frac{-44}{125}\right).$$

$$4\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{336}{625}, \frac{-527}{625}\right).$$

$(x_1, y_1) + (0, 1) = (x_1, y_1)$.

$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}$.

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$
$= \{0, 1, 2, 3, -3, -2, -1\}$
with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

of clock addition:

5:00”

$1/2) + (1/2, -\sqrt{3/4})$

$-\sqrt{3/4}) =$ “7:00”.

9:00”

$\sqrt{3/4}) + (-1, 0)$

$1/2) =$ “2:00”.

$= \left(\frac{24}{25}, \frac{7}{25}\right).$

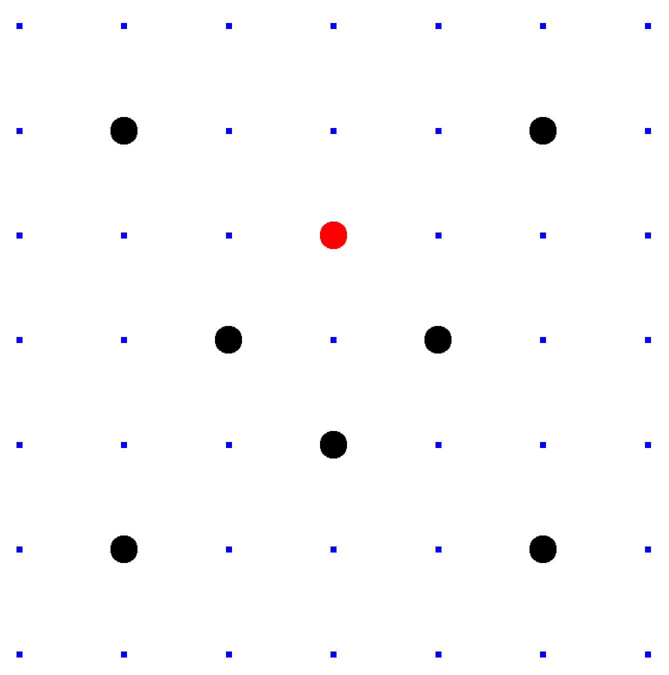
$= \left(\frac{117}{125}, \frac{-44}{125}\right).$

$= \left(\frac{336}{625}, \frac{-527}{625}\right).$

$(0, 1) = (x_1, y_1).$

$(-x_1, y_1) = (0, 1).$

<u>Clocks over finite fields</u>



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}.$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x
...   for
...     if
...
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

tion:

$-\sqrt{3/4})$

7:00".

$1, 0)$

0".

.

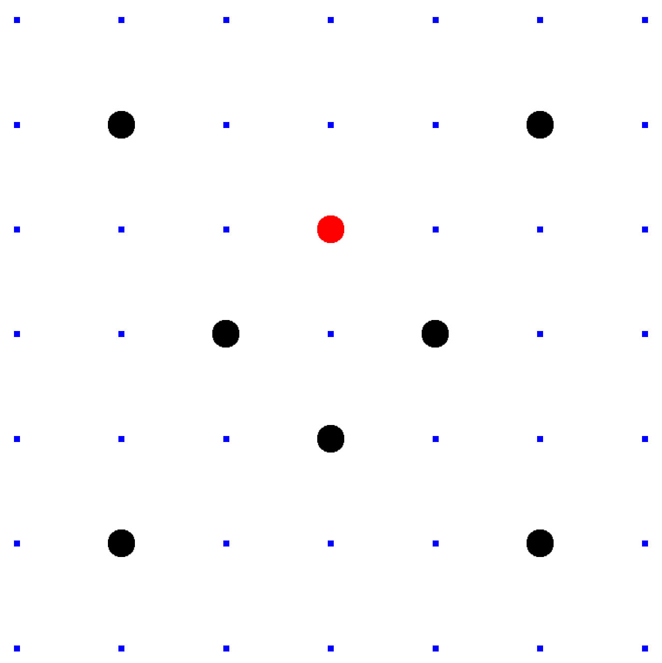
$\frac{4}{5}\Big)$.

$\frac{27}{5}\Big)$.

$y_1)$.

$(0, 1)$.

Clocks over finite fields



$\mathsf{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}$.

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

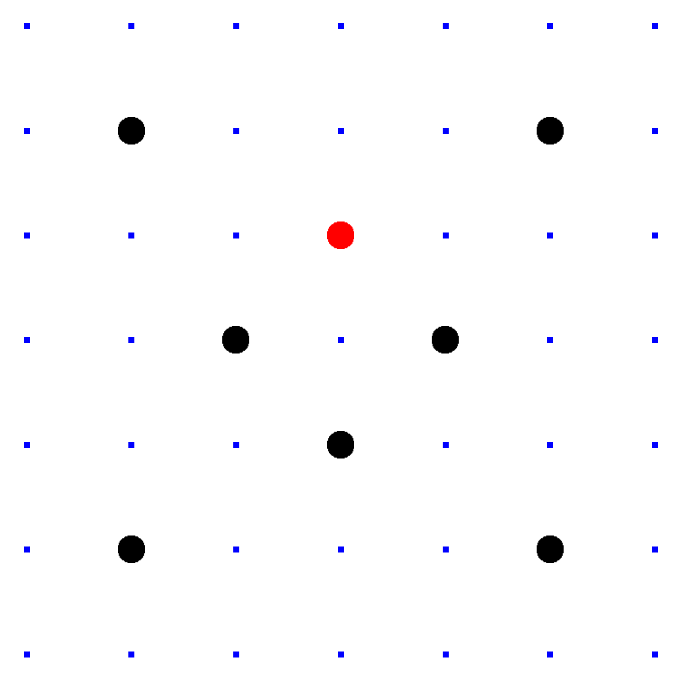
$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7)
...   for y in range(
...     if (x*x+y*y)
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

Clocks over finite fields



$\mathrm{Clock}(\mathbf{F}_7) = \left\{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2{+}y^2{=}1\right\}.$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

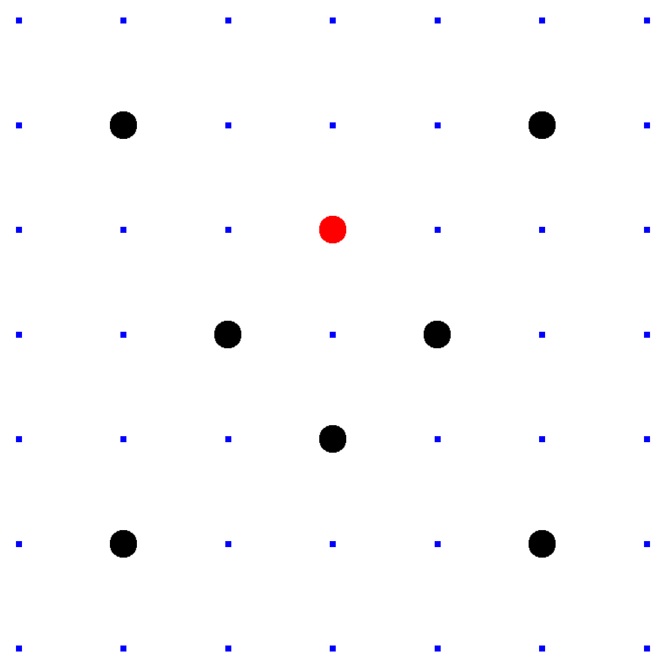
$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

Clocks over finite fields



$\mathrm{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}$.

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

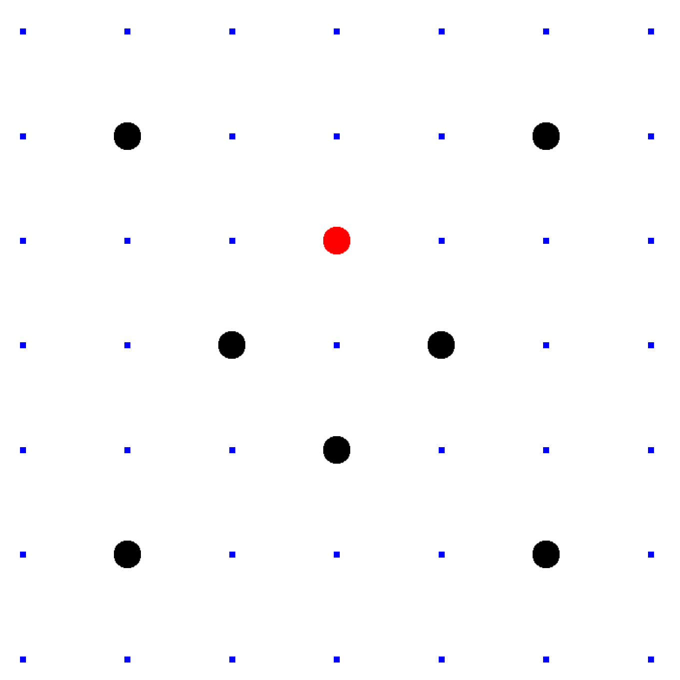
$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

r finite fields



$= \{(x,y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}$.

$\{0,1,2,3,4,5,6\}$

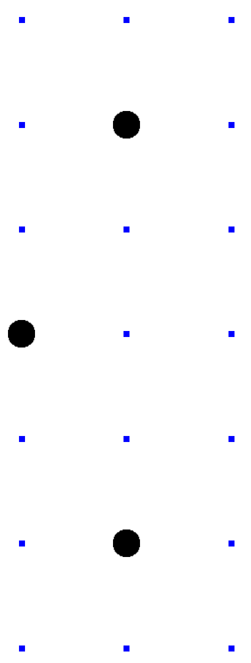
$\{0,1,2,3,-3,-2,-1\}$

netic modulo 7.

$= 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class
...   def
...     se
...   def
...     re
...   __re
...
>>> print
2
>>> print
6
>>> print
0
>>> print
3
```



$\mathbf{F}_7 \times \mathbf{F}_7 : x^2{+}y^2{=}1\}$.

$5, 6\}$

$3, -2, -1\}$

$7.$

$= 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(se
...     self.int = x
...   def __str__(sel
...     return str(se
...   __repr__ = __st
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

$+y^2=1\}.$

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
 in range(7):
 y in range(7):
f (x*x+y*y) % 7 == 1:
 print (x,y)
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__e
...   lamb
>>>
>>> print
True
>>> print
True
>>> print
True
>>> print
False
>>> print
False
>>> print
False
```

```
):
(7):
 % 7 == 1:
)
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.i
>>>
>>> print F7(7) == F7
True
>>> print F7(10) == F
True
>>> print F7(-3) == F
True
>>> print F7(0) == F7
False
>>> print F7(0) == F7
False
>>> print F7(0) == F7
False
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
F7:
__init__(self,x):
elf.int = x % 7
__str__(self):
eturn str(self.int)
epr__ = __str__

F7(2)

F7(6)

F7(7)

F7(10)
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__a
...   lamb
>>> F7.__s
...   lamb
>>> F7.__m
...   lamb
>>>
>>> print
0
>>> print
4
>>> print
3
>>>
```

```
elf,x):
 % 7
lf):
elf.int)
tr__
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(
>>> F7.__sub__ = \
...   lambda a,b: F7(
>>> F7.__mul__ = \
...   lambda a,b: F7(
>>>
>>> print F7(2) + F7(
0
>>> print F7(2) - F7(
4
>>> print F7(2) * F7(
3
>>>
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.i
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.i
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.i
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

```
eq__ = \
bda a,b: a.int == b.int

 F7(7) == F7(0)

 F7(10) == F7(3)

 F7(-3) == F7(4)

 F7(0) == F7(1)

 F7(0) == F7(2)

 F7(0) == F7(3)
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger exa

```
p = 10000
class Fp:
  ...

def clocka
  x1,y1 =
  x2,y2 =
  x3 = x1*
  y3 = y1*
  return x
```

```
int == b.int

7(0)

F7(3)

F7(4)

7(1)

7(2)

7(3)
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger example: Clock(

```
p = 1000003
class Fp:
  ...

def clockadd(P1,P2):
  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger example: Clock($\mathbf{F}_{1000003}$).

```
p = 1000003
class Fp:
  ...

def clockadd(P1,P2):
  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
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>>> print F7(2) + F7(5)
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4
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3
>>>
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  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
add__ = \
bda a,b: F7(a.int + b.int)
sub__ = \
bda a,b: F7(a.int - b.int)
mul__ = \
bda a,b: F7(a.int * b.int)

 F7(2) + F7(5)

 F7(2) - F7(5)

 F7(2) * F7(5)
```

Larger example: Clock($\mathbf{F}_{1000003}$).

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  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (F
>>> P2 = c
>>> print
(4000, 7)
>>> P3 = c
>>> print
(15000, 26
>>> P4 = c
>>> P5 = c
>>> P6 = c
>>> print
(780000, 1
>>> print
(780000, 1
>>>
```

```
(a.int + b.int)

(a.int - b.int)

(a.int * b.int)

(5)

(5)

(5)
```

Larger example: Clock($\mathbf{F}_{1000003}$).

```
p = 1000003
class Fp:
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  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (Fp(1000),Fp(
>>> P2 = clockadd(P,P
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,
>>> P5 = clockadd(P4,
>>> P6 = clockadd(P5,
>>> print P6
(780000, 1351)
>>> print clockadd(P3
(780000, 1351)
>>>
```

nt)

nt)

nt)

Larger example: Clock($\mathbf{F}_{1000003}$).

```
p = 1000003
class Fp:
  ...

def clockadd(P1,P2):
  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

Larger example: Clock($\mathbf{F}_{1000003}$).

```
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  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (Fp(1000),Fp(2))
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(4000, 7)
>>> P3 = clockadd(P2,P)
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(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

mple: Clock($\mathbf{F}_{1000003}$).

```
03
```

```
add(P1,P2):
 P1
 P2
*y2+y1*x2
*y2-x1*x2
x3,y3
```

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def sc
...   if n
...   if n
...   Q =
...   Q =
...   if n
...   retu
...
>>> n = ou
>>> scalar
(947472, 7
>>>
```

Can you fig

$(\mathbf{F}_{1000003})$.

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,
...   if n == 0: retu
...   if n == 1: retu
...   Q = scalarmult(
...   Q = clockadd(Q,
...   if n % 2: Q = c
...   return Q
...
>>> n = oursixdigitse
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),F
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
...   Q = clockadd(Q,Q)
...   if n % 2: Q = clockadd(P,Q
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
...   Q = clockadd(Q,Q)
...   if n % 2: Q = clockadd(P,Q)
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

```
Fp(1000),Fp(2))
clockadd(P,P)
 P2

clockadd(P2,P)
 P3
6)
clockadd(P3,P)
clockadd(P4,P)
clockadd(P5,P)
 P6
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 clockadd(P3,P3)
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```

```
>>> def scalarmult(n,P):
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...   Q = clockadd(Q,Q)
...   if n % 2: Q = clockadd(P,Q)
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

Clock cryp

The "Clock

Standardiz
and **base p**

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Bob compu

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to encrypt

```
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,P)
,P)
,P)
3,P3)
```

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
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...   if n % 2: Q = clockadd(P,Q)
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

Clock cryptography

The “Clock Diffie–Hell

Standardize a large prin
and **base point** (x, y)

Alice chooses big secret
Alice computes her pub

Bob chooses big secret
Bob computes his publi

Alice computes $a(b(x, y$
Bob computes $b(a(x, y$
They use this shared se
to encrypt with AES-G

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
...   Q = clockadd(Q,Q)
...   if n % 2: Q = clockadd(P,Q)
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

Clock cryptography

The "Clock Diffie–Hellman protocol

Standardize a large prime p
and **base point** $(x, y) \in \text{Clock}(\mathbf{F}_p)$.

Alice chooses big secret a.
Alice computes her public key $a(x,$

Bob chooses big secret b.
Bob computes his public key $b(x, y)$

Alice computes $a(b(x, y))$.
Bob computes $b(a(x, y))$.
They use this shared secret
to encrypt with AES-GCM etc.

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
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...
>>> n = oursixdigitsecret
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(947472, 736284)
>>>
```

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```
calarmult(n,P):
n == 0: return (Fp(0),Fp(1))
n == 1: return P
 scalarmult(n//2,P)
 clockadd(Q,Q)
n % 2: Q = clockadd(P,Q)
urn Q


ursixdigitsecret
rmult(n,P)
736284)
```

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```
,P):
urn (Fp(0),Fp(1))
urn P
(n//2,P)
,Q)
clockadd(P,Q)



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```

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Bob computes his public key $b(x, y)$.

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Bob computes $b(a(x, y))$.
They use this shared secret
to encrypt with AES-GCM etc.

Alice's secret key a

Alice's public key
$a(x, y)$

{Alice, Bob}'s
shared secret
$ab(x, y)$

=

```
Fp(1))
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Q)
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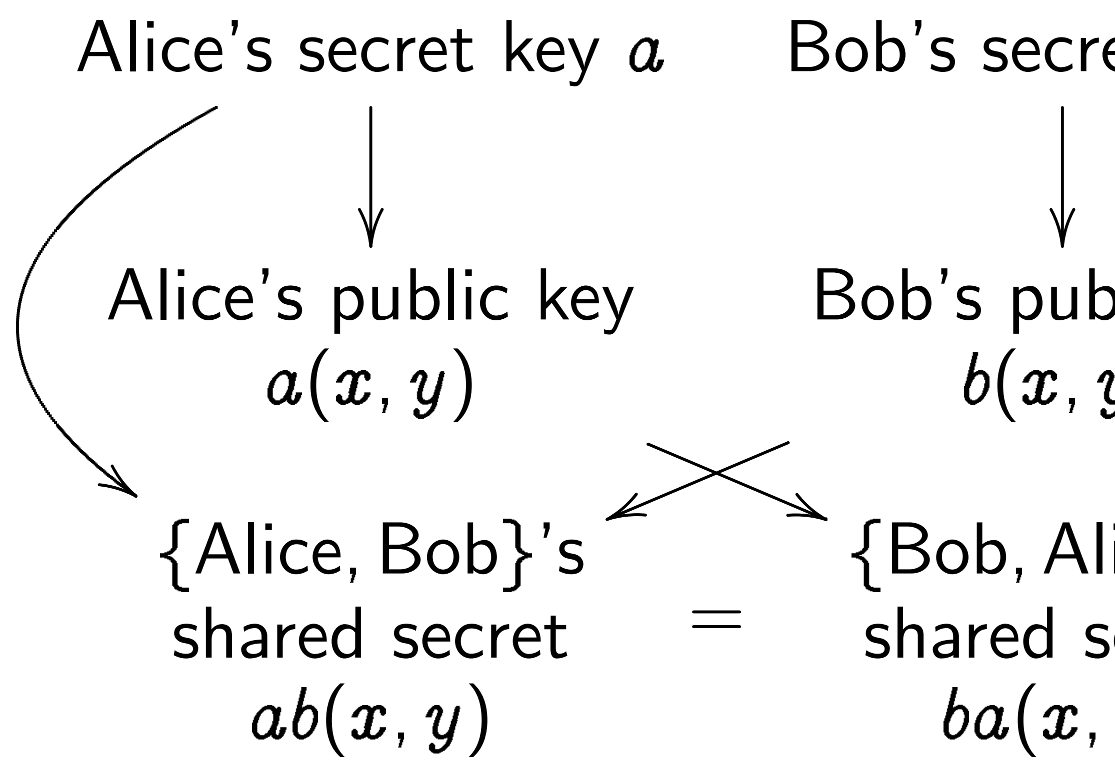

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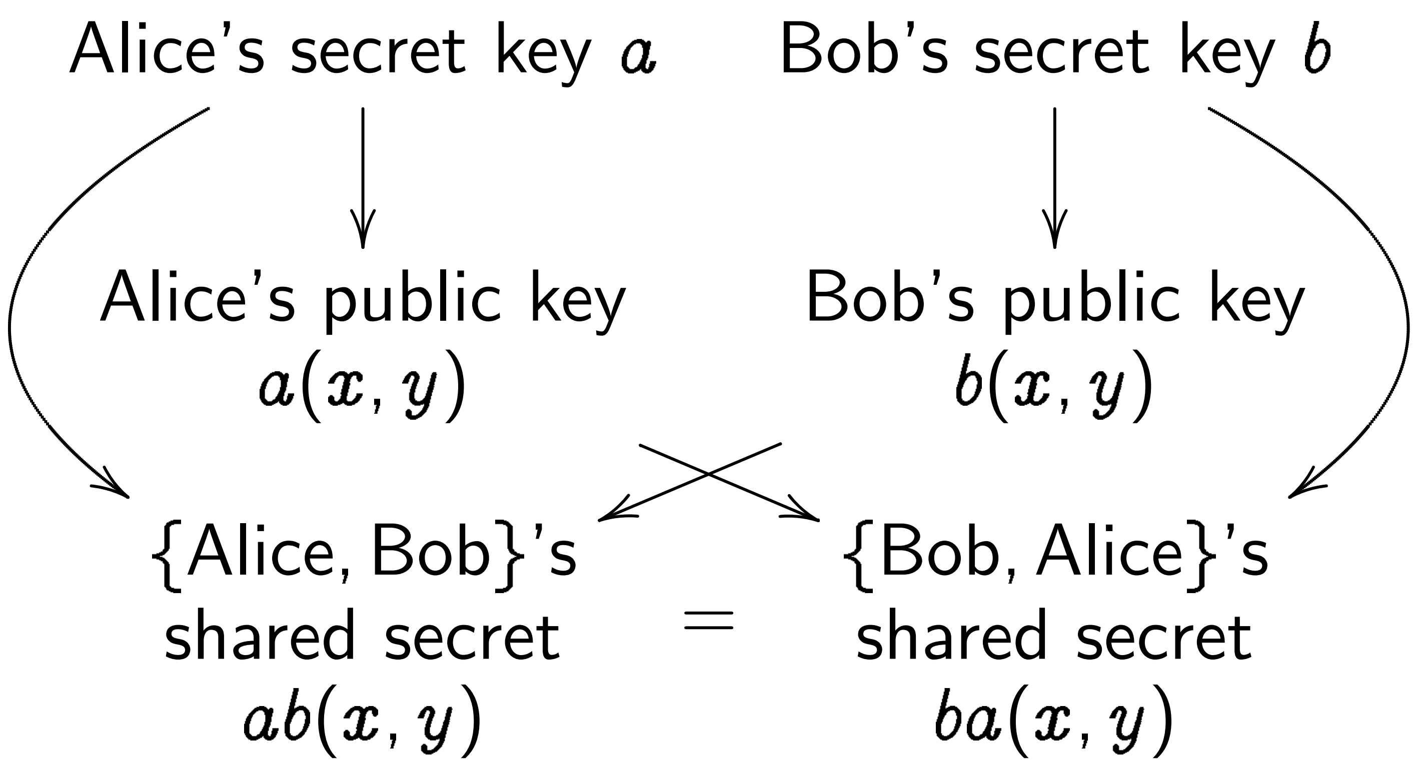

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Bob chooses big secret b.
Bob computes his public key $b(x, y)$.

Alice computes $a(b(x, y))$.
Bob computes $b(a(x, y))$.
They use this shared secret
to encrypt with AES-GCM etc.

Alice's secret key a | Bob's secret key b

Alice's public key $a(x, y)$ | Bob's public key $b(x, y)$

{Alice, Bob}'s shared secret $ab(x, y)$ $=$ {Bob, Alice}'s shared secret $ba(x, y)$

Warning #1: Many choices of p are unsafe!

Warning #2: Clocks aren't elliptic!
Can use index calculus
to attack clock cryptography.
To match RSA-3072 security
need $p \approx 2^{1536}$.

tography

k Diffie–Hellman protocol":

e a large prime p

oint $(x, y) \in \text{Clock}(\mathbf{F}_p)$.

ses big secret a.

putes her public key $a(x, y)$.

es big secret b.

utes his public key $b(x, y)$.

putes $a(b(x, y))$.

utes $b(a(x, y))$.

his shared secret

with AES-GCM etc.

$$
\begin{tikzcd}
\text{Alice's secret key } a \arrow[d] \arrow[dd, bend right] & & \text{Bob's secret key } b \arrow[d] \arrow[dd, bend left] \\
\begin{array}{c}\text{Alice's public key}\\ a(x,y)\end{array} \arrow[drr] & & \begin{array}{c}\text{Bob's public key}\\ b(x,y)\end{array} \arrow[dll] \\
\begin{array}{c}\{\text{Alice, Bob}\}\text{'s}\\ \text{shared secret}\\ ab(x,y)\end{array} & = & \begin{array}{c}\{\text{Bob, Alice}\}\text{'s}\\ \text{shared secret}\\ ba(x,y)\end{array}
\end{tikzcd}
$$

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Warning #

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Alice's secret key a Bob's secret key b

Alice's public key $a(x, y)$

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the public keys $a(x, y)$

Attacker sees how much
Alice uses to compute
Often attacker can see
for *each operation* perf
not just total time.
This reveals secret scal

Some timing attacks: 2
2013 "Lucky Thirteen"
2014 Benger–van de Po

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Alice's secret key a Bob's secret key b

Alice's public key $a(x, y)$ Bob's public key $b(x, y)$

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the public keys $a(x, y)$ and $b(x, y)$.

Attacker sees how much *time*
Alice uses to compute $a(b(x, y))$.
Often attacker can see time
for *each operation* performed by Ali
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This reveals secret scalar a.

Some timing attacks: 2011 Brumley
2013 "Lucky Thirteen" (not ECC);
2014 Benger–van de Pol–Smart–Yar

Alice's secret key a Bob's secret key b

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Fix: **constant-time** code,
performing same operations
no matter what scalar is.

ecret key a Bob's secret key b

public key Bob's public key

$x, y)$ $b(x, y)$

, Bob}'s {Bob, Alice}'s

d secret $=$ shared secret

$x, y)$ $ba(x, y)$

1: Many choices of p are unsafe!

2: Clocks aren't elliptic!

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clock cryptography.

RSA-3072 security

^{1536}.

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Addition o

$x^2 + y^2 =$

Sum of $(x_1$

$((x_1y_2+y_1$

$(y_1y_2-x_1$

Bob's secret key b

Bob's public key
$b(x, y)$

{Bob, Alice}'s
shared secret
$ba(x, y)$

oices of p are unsafe!

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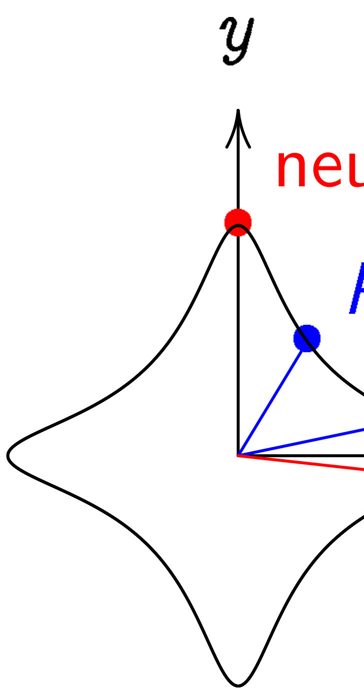

$x^2 + y^2 = 1 - 30x^2y^2$.
Sum of (x_1, y_1) and $(x$
$((x_1y_2{+}y_1x_2)/(1{-}30x$
$(y_1y_2{-}x_1x_2)/(1{+}30x$

et key b

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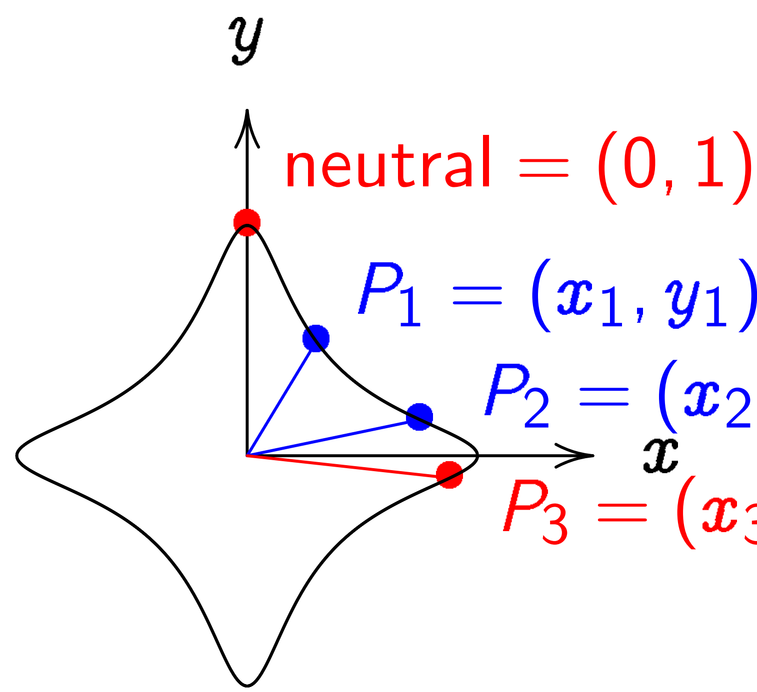
Fix: **constant-time** code, performing same operations no matter what scalar is.

Addition on an elliptic curve



$x^2 + y^2 = 1 - 30x^2y^2$.
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Warning #3: Attacker sees more than the public keys $a(x, y)$ and $b(x, y)$.

Attacker sees how much *time* Alice uses to compute $a(b(x, y))$. Often attacker can see time for *each operation* performed by Alice, not just total time. This reveals secret scalar a.

Some timing attacks: 2011 Brumley–Tuveri; 2013 "Lucky Thirteen" (not ECC); 2014 Benger–van de Pol–Smart–Yarom; etc.

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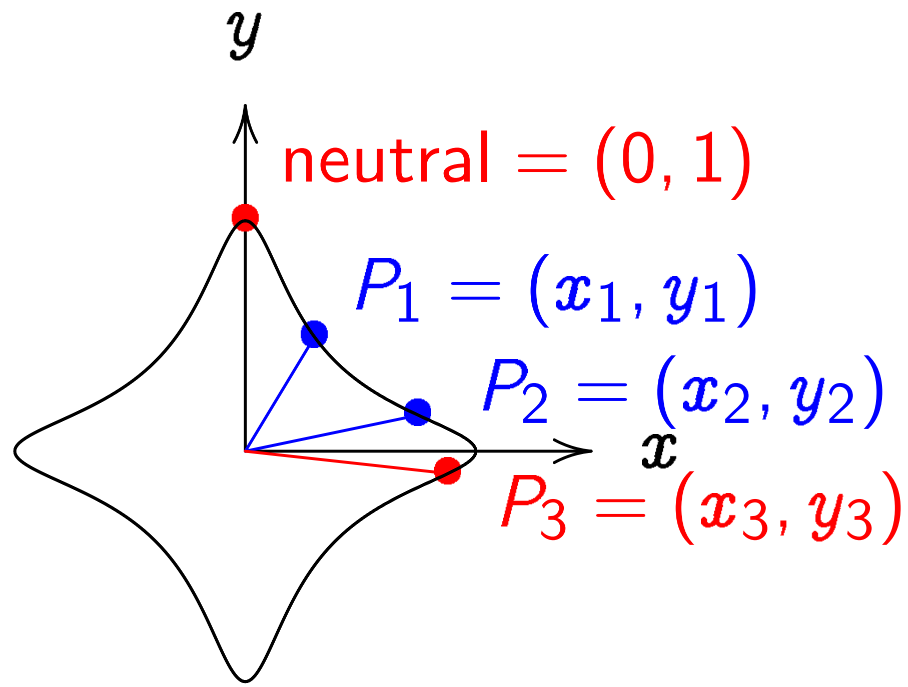

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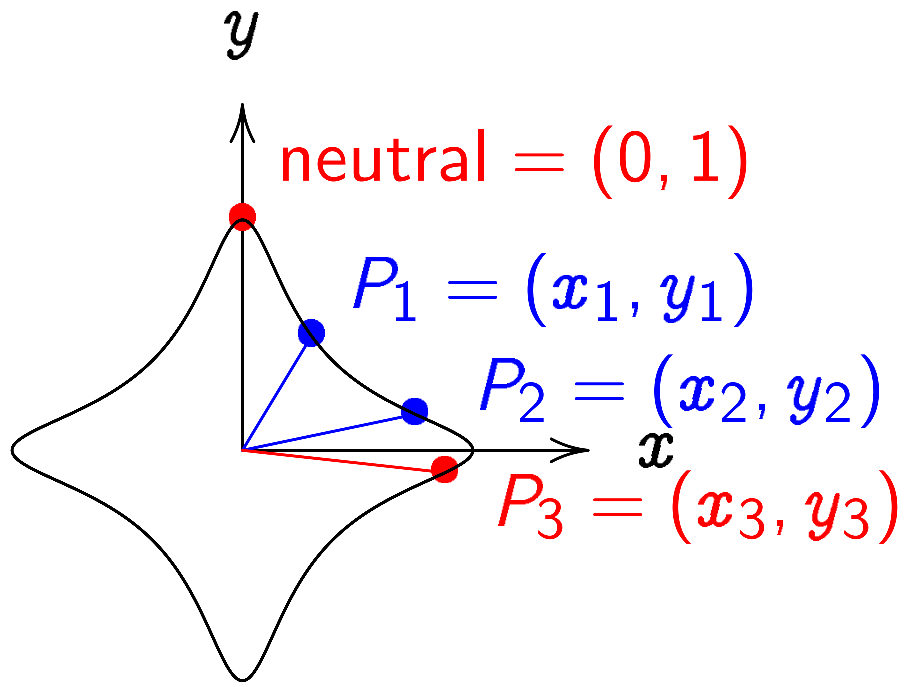

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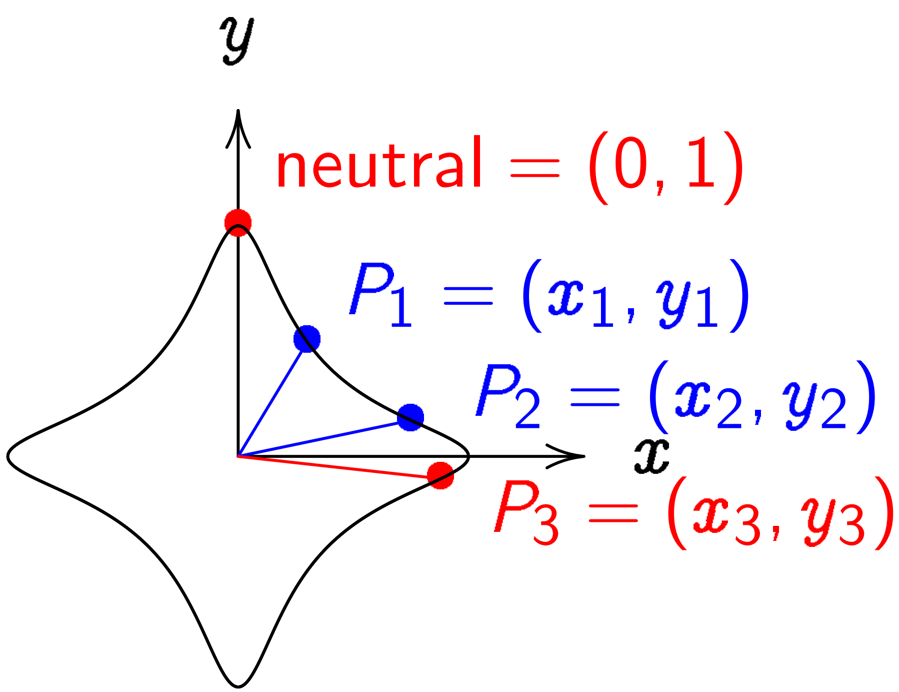

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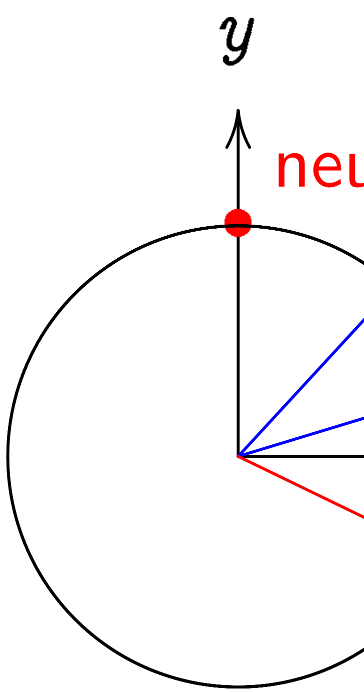

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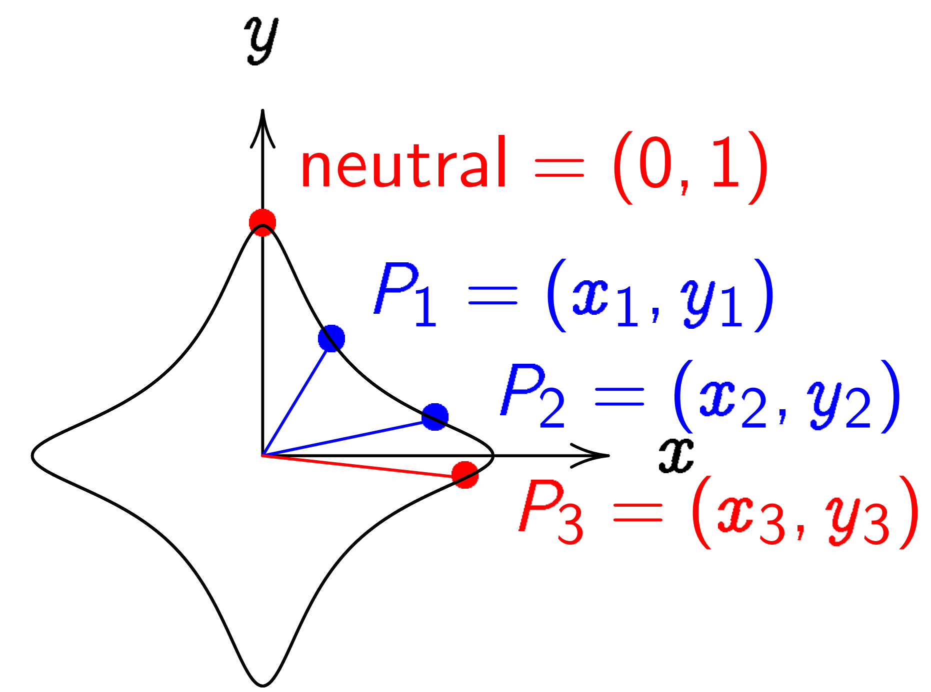

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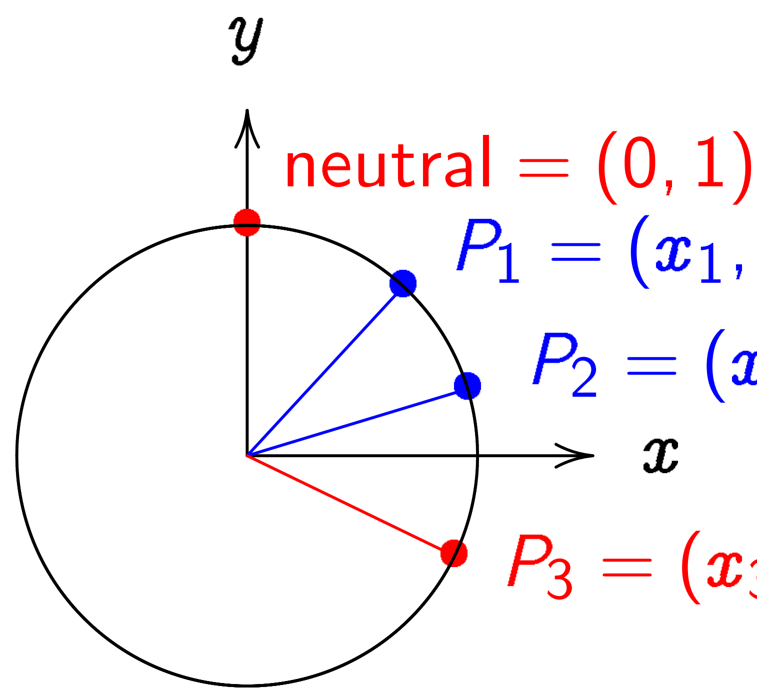

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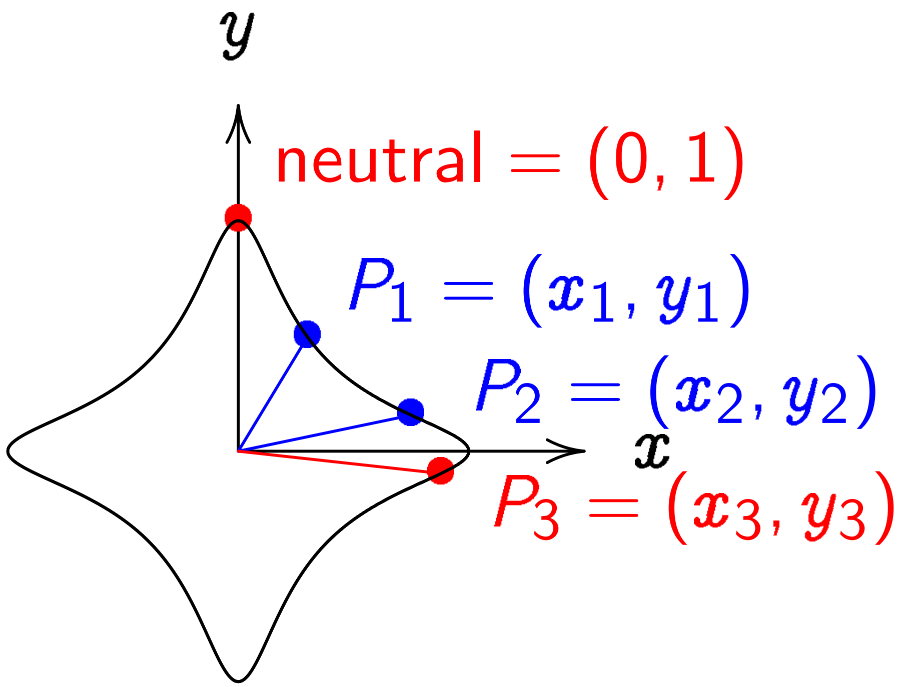

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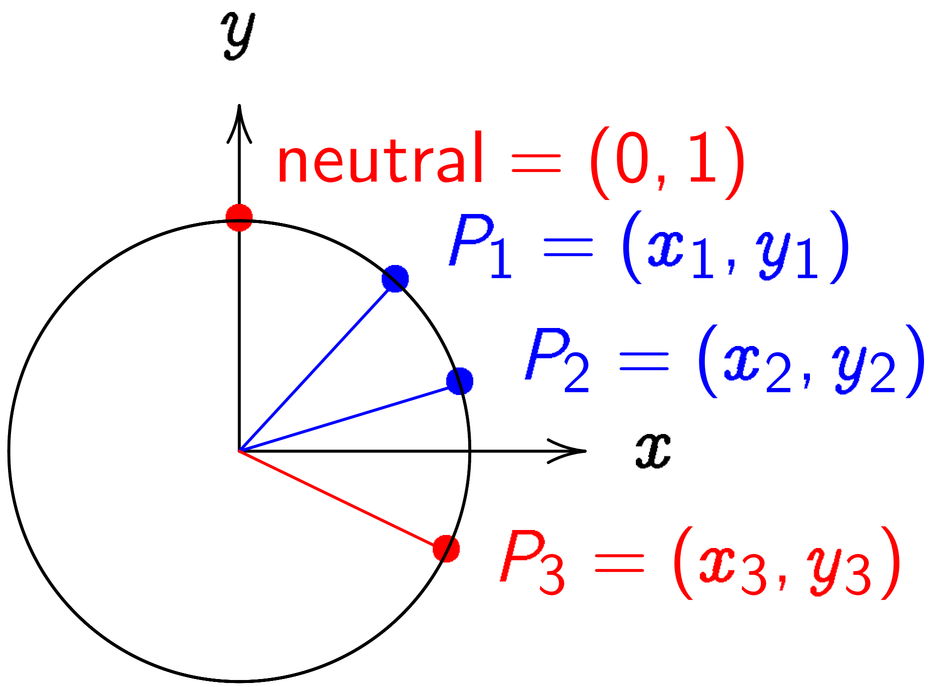

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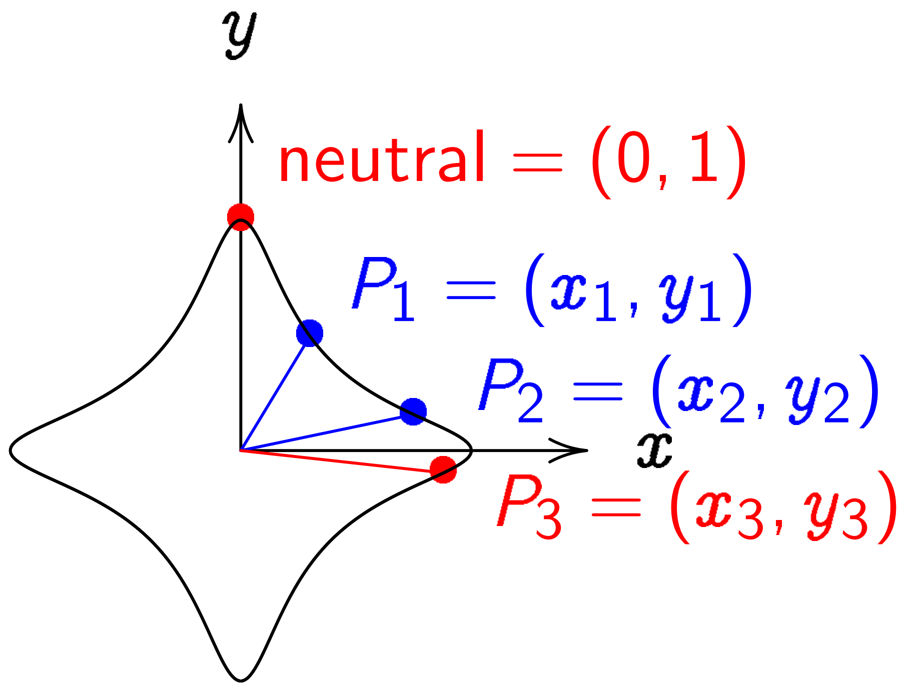

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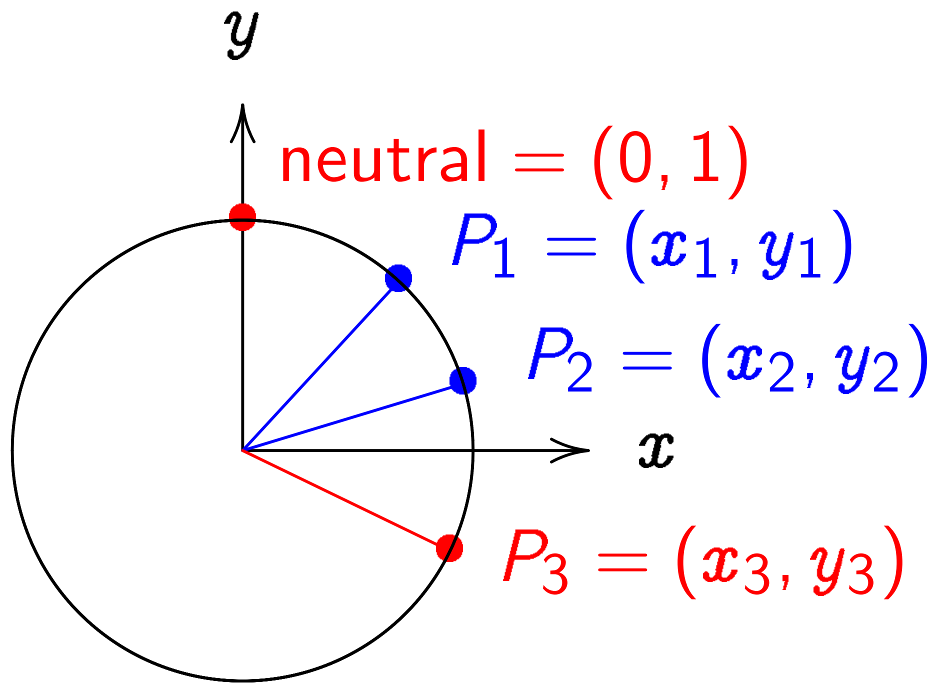

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  x3 = (x1
  y3 = (y1
  return x
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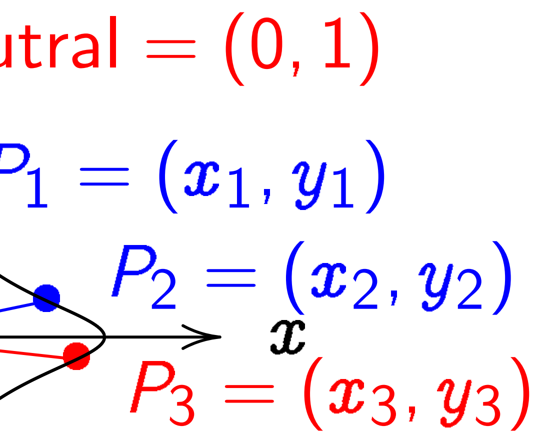

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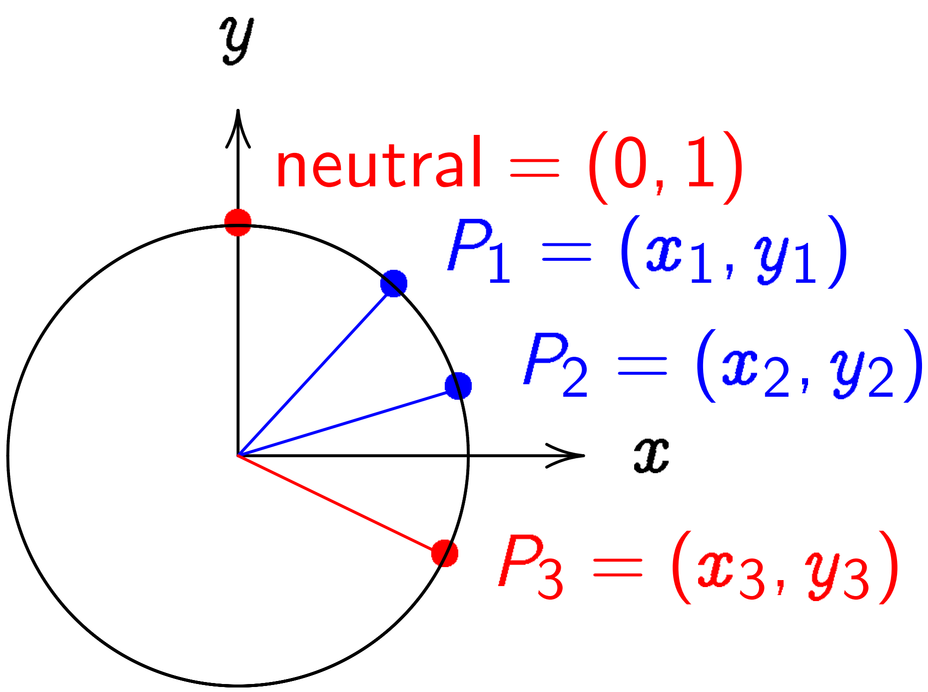

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More elliptic curves

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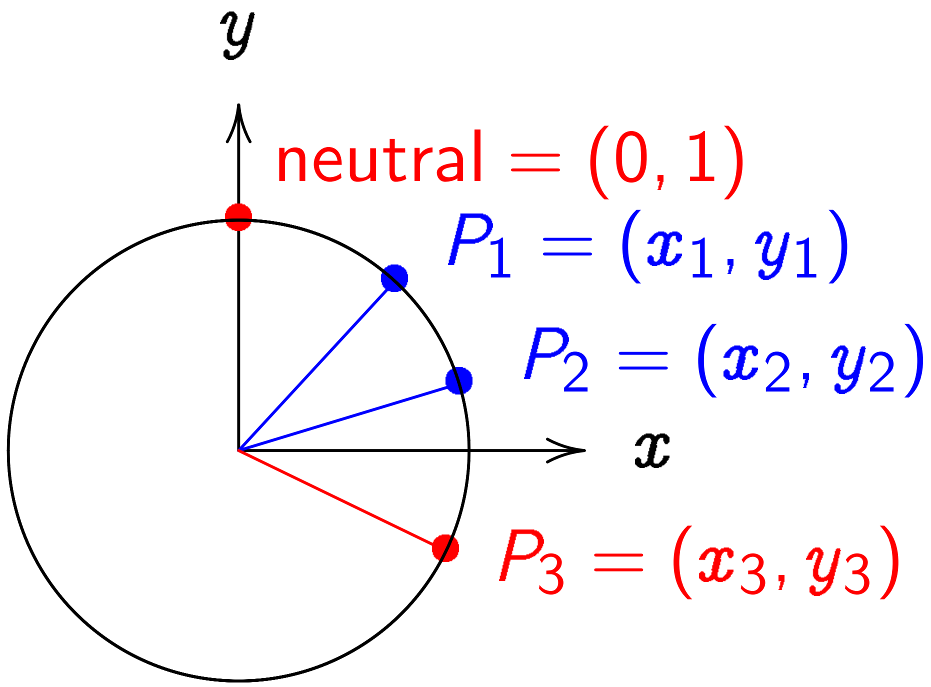

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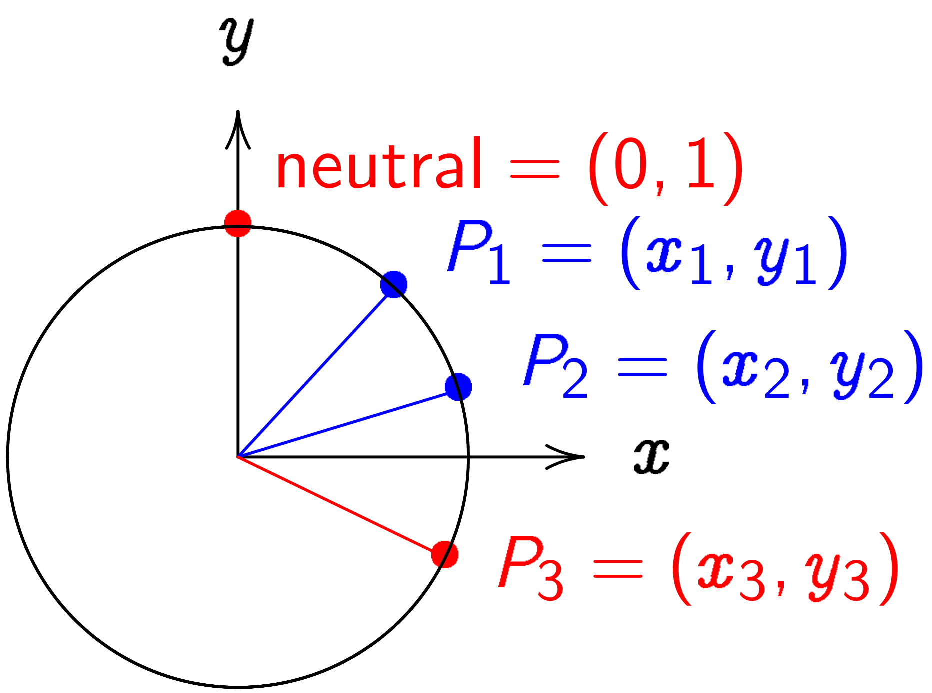

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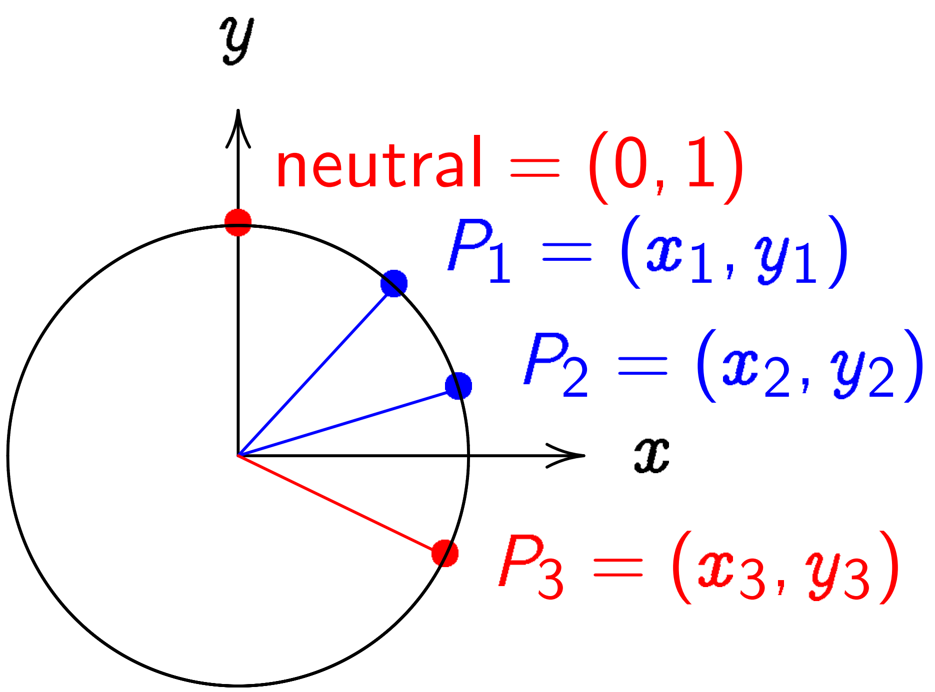

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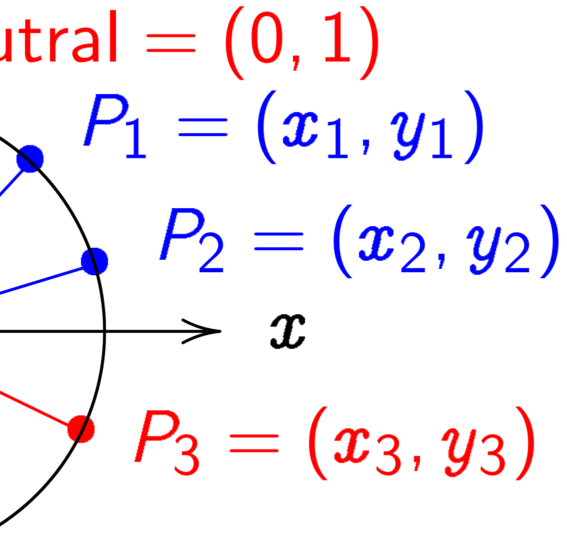

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Store (X, Y, Z) representing $(X/Z, Y/Z)$.

Another option: "extended coordinates".
Store projective (X, Y, Z) and $T = XY/Z$.

See "Explicit Formulas Database"
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Elliptic-curve cryptography

Standardize prime p, safe non-squar
base point (x, y) on elliptic curve.

Alice knows her secret key a
and Bob's public key $b(x, y)$.
Alice computes (and caches)
shared secret $ab(x, y)$.

Alice uses shared secret to encrypt
and authenticate packet for Bob.

Packet overhead at high security lev
32 bytes for Alice's public key,
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See "Explicit Formulas Database"
for many more options and speedups:
hyperelliptic.org/EFD

Elliptic-curve cryptography

Standardize prime p, safe non-square d,
base point (x, y) on elliptic curve.

Alice knows her secret key a
and Bob's public key $b(x, y)$.
Alice computes (and caches)
shared secret $ab(x, y)$.

Alice uses shared secret to encrypt
and authenticate packet for Bob.

Packet overhead at high security level:
32 bytes for Alice's public key,
24 bytes for nonce,
16 bytes for authenticator.

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A safe example

Choose $p = 2^{255} - 19$.
Choose $d = 121665/12$
this is non-square in $\mathbf{F}_p$

$x^2 + y^2 = 1 + dx^2y^2$
is a safe curve for ECC.

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def scala
  x2,z2,x
  for i i
    bit =
    x2,x3
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    x3,z3

    x2,z2


    x2,x3
    z2,z3
  return
```

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$= (x'-1)/(x'+1)$.

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Much nicer than Weiers
curves with the "Montg

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x
  for i in reversed(r
    bit = 1 & (n >> i
    x2,x3 = cswap(x2,
    z2,z3 = cswap(z2,
    x3,z3 = ((x2*x3-z
            x1*(x2*z3
    x2,z2 = ((x2^2-z2
            4*x2*z2*(
    x2,x3 = cswap(x2,
    z2,z3 = cswap(z2,
  return x2*z2^(p-2)
```

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Much nicer than Weierstrass: Mont
curves with the "Montgomery ladde

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbi
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
    x3,z3 = ((x2*x3-z2*z3)^2,
             x1*(x2*z3-z2*x3)^2)
    x2,z2 = ((x2^2-z2^2)^2,
             4*x2*z2*(x2^2+A*x2*z
    x2,x3 = cswap(x2,x3,bit)
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  return x2*z2^(p-2)
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Much nicer than Weierstrass: Montgomery curves with the “Montgomery ladder”.

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbits)):
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
    x3,z3 = ((x2*x3-z2*z3)^2,
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    x2,z2 = ((x2^2-z2^2)^2,
             4*x2*z2*(x2^2+A*x2*z2+z2^2))
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
  return x2*z2^(p-2)
```

n Weierstrass curves

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Much nicer than Weierstrass: Montgomery curves with the "Montgomery ladder".

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbits)):
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
    x3,z3 = ((x2*x3-z2*z3)^2,
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You can pick any of these standards

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Example of criterion in all standards
Standard base point (x, y)
has huge prime "order" ℓ,
i.e., exactly ℓ different multiples.

All criteria are computer-verifiable.
See our evaluation site for scripts:
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The attacker sent you (

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`1e86bec6c6bac120535e`

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`d123d55f68100099b65a`

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for $x_1 \neq x_2$, $(x_1, y_1) + (x_2, y_2)$
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$\lambda = (y_2 - y_1)/(x_2 - x_1)$;
for $y_1 \neq 0$, $(x_1, y_1) + (x_1, y_1) =$
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Addition on Weierstrass curves
$y^2 = x^3 + a_4 x + a_6$:
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(x_3, y_3) with $x_3 = \lambda^2 - x_1 - x_2$,
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$y' =$ `0d124e9e94dcede52aa0e3bcac1852cfed28eb86039c0d8e0cfaa4ae703eac07`,

a point of order 19559 on $y^2 = x^3 - 3x + 211$; learns your secret a mod 19559.

Etc. Uses “Chinese remainder theorem” to combine this information.

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Never send uncompressed (x, y).
Design protocols to compress
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Drastically limits possibilities
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Always multiply DH scalar by cof

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Design DH protocols to multiply by

Always choose twist-secure curve

Montgomery formulas use only A, but modifying B gives only *two* diff curve orders. Require both of these to be large primes times small cofac

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2013.09 Douglas Stebila: Reasons to support Curve25519 are "efficiency and resistance to side-channel attacks" rather than concerns about backdoors.

2013.09 Nick Mathewson: "In the FOSS cryptography world nowadays, I see many more new users of curve25519 than of the NIST curves, because of efficiency and ease-of-implementation issues."

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Sage scripts to verify criteria for ECDLP security and ECC security: `safecurves.cr.yp.to`

Analysis of manipulability of various curve-generation methods: `safecurves.cr.yp.to/bada55.html`

Many computer-verified addition formulas: `hyperelliptic.org/EFD/`

Python scripts for this talk: `ecchacks.cr.yp.to`